

# MICROSTRIP PATCH ANTENNA ARRAY WITH LMS ADAPTIVE ALGORITHM FOR 2.4 GHZ WIRELESS COMMUNICATION SYSTEMS

**Alejandro Iturri-Hinojosa, Cirilo G. León-Vega Mohamed Badaoui**

Instituto Politécnico Nacional, ESIME Zacatenco, C.P. 07320 Ciudad de México, México.

## ABSTRACT

The design and simulation of microstrip patch antenna array with Least Mean Square (LMS) adaptive algorithm is presented. The antenna design is oriented for 2.4 GHz wireless communication system. LMS is a gradient based algorithm that was studied recently to adaptively track incident signals from satellites and its application on wireless communication systems. This adaptive technique is based on minimization of mean-square error of the LMS algorithm. The calculated variable weights are introduced in order to automatically steer the main beam to a desired direction and reject interfering signals from specific directions. The LMS algorithm is used to obtain the corresponding weights for the  $11 \times 11$  element planar microstrip array designed for the operation frequency of 2.4 GHz. The desired signal direction is set to  $30^\circ$  with an interfering signal at  $-20^\circ$ , both on H-Plane radiation.

**Key words:** Microstrip patch antenna, planar microstrip array, LMS adaptive algorithm and 2.4 GHz wireless communication systems.

## INTRODUCTION

The ability to cancel the noise and interfering signals is the main interest in adaptive array systems. Also, there is great interest in the reduction of multipath effect and fading. Microstrip antenna array have attracted widespread attention due to their small size, light weight, low cost, easily integrated with circuits, are very versatile and are suitable for applications requiring pencil beams.

### Rectangular microstrip geometry

A microstrip element consists of a conducting patch impressed over a substrate with dielectric constant ( $\epsilon_r$ ), its geometry is depicted in Figure 1. This type of microstrip uses an inset feed microstrip line.

The strip width ( $W$ ) is calculated with equation (1):

$$W = \frac{\lambda_0}{2} \sqrt{\frac{2}{\epsilon_r + 1}} \quad (1)$$

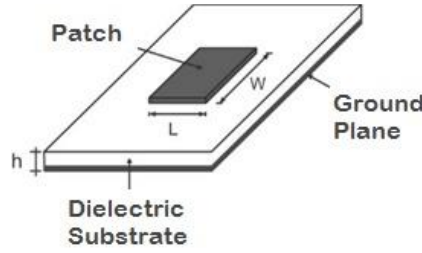
Where  $\lambda_0$  is the free-space wavelength ( $\lambda_0 = \frac{c}{f}$ ).

The effective dielectric constant, which depends on the dielectric constant of the substrate material and the physical dimensions of the microstrip line, is calculated using the following equation.

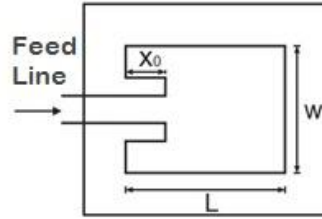
$$\epsilon_{ef} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left( \frac{1}{\sqrt{1 + \frac{10h}{W}}} \right) \quad (2)$$

The patch length (L) is calculated with equation (3):

$$L = L_{ef} - 2\Delta L \quad (3)$$



(a)



(b)

**Figure I. a) Microstrip line feed element and its b) geometry (Balanis 2016, Leijja 2014)**

where  $L_{ef}$  is the effective length and  $\Delta L$  is the incremental length. Both are calculated with equations (4) and (5), respectively.

$$L_{ef} = \frac{\lambda_g}{2} - 2\Delta L \quad (4)$$

$$\Delta L = 0.412h \left[ \frac{\epsilon_{ef} + 0.3}{\epsilon_{ef} - 0.258} \right] \left[ \frac{\frac{W}{h} + 0.264}{\frac{W}{h} + 0.813} \right] \quad (5)$$

And the wavelength  $\lambda_g$  is:

$$\lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_{ef}}} \quad (6)$$

The resonant frequency can be evaluated using equation (7):

$$f_r = \frac{1}{2\sqrt{\mu_0\epsilon_0}(L+2\Delta L)\sqrt{\epsilon_{ef}}} \quad (7)$$

where  $\mu_0$  is the permeability of free space, and  $\epsilon_0$  its permittivity.

The microstrip antenna radiates perpendicular to the ground plane and has a wide beamwidth. In order to obtain the radiated electric field in spherical coordinates, equation (8) for the element factor is used (Leija 2014).

$$E_{\varphi}^t = +j \frac{k_0 h W E_0 e^{-jk_0 r}}{\pi r} \left\{ \sin \theta \frac{\sin(X)}{X} \frac{\sin(Z)}{Z} \right\} \cdot \cos \left( \frac{k_0 L_e}{2} \sin \theta \sin \varphi \right) \quad (8)$$

where:

$$X = \frac{k_0 h}{2} \sin \theta \cos \varphi,$$

$$Z = \frac{k_0 W}{2} \cos \theta,$$

and  $k_0 \left( = \frac{2\pi}{\lambda_0} \right)$  is the wave number in free space propagation.

The length  $X_0$  of the inset microstrip line feed is calculated with equation (9):

$$X_0 = \frac{\cos^{-1}\{[4R_{in}(G_1 G_{12})] - 1\}L}{2\pi} \quad (9)$$

where the conductances  $G_1$  and  $G_{12}$  are equal to:

$$G_1 = \frac{1}{120\pi^2} \int_0^\pi \left[ \frac{\frac{k_0 W}{2} \cos \theta}{\cos \theta} \right]^2 \sin^3 \theta d\theta \quad (10)$$

$$G_{12} = \frac{1}{120\pi^2} \int_0^\pi \left[ \frac{\frac{k_0 W}{2} \cos \theta}{\cos \theta} \right]^2 J_0(k_0 L \sin \theta) \cdot \sin^3 \theta d\theta \quad (11)$$

$J_0$  is the Bessel function of first order, defined by:

$$J_0(X) = 1 - \frac{X^2}{2^2} + \frac{X^4}{2^2 4^2} - \frac{X^6}{2^2 4^2 6^2} + \dots$$

In this case  $X = k_0 L \sin \theta$ .

In order to have the line feed coupled to the microstrip, both must be of  $50 \Omega$  of impedance.

### Microstrip element for 2.4 GHz of frequency

In the design of the microstrip element geometry, equations (1) to (11) are used, taking into account the characteristics of the materials for the operating frequency of 2.4 GHz.

The relative permeability of 4.6 of FR4 PCB substrate is considered as patch support. The dielectric loss tangent is 0.022 with 1.6 mm high of substrate.

Table 1 summarizes the rectangular microstrip geometry for 2.4 GHz of frequency.

**Table I. Rectangular microstrip geometry for 2.4 GHz**

Parameter	Value
$W$	37.40 mm
$\epsilon_{eff}$	4.26
$\epsilon_r$	4.60
$L$	28.70 mm
$f_r$	2.38 GHz
$G_1$	$9.36 \times 10^{-4} \Omega^{-1}$
$G_{12}$	$5.80 \times 10^{-4} \Omega^{-1}$
$X_0$	12.50 mm
$Y_0$	1.90 mm

### Planar microstrip array antenna

A number of similar elements of Figure 1 in a rectangular shape may be used to produce an adaptive beam behavior. The LMS algorithm could be implemented in the planar microstrip array antenna, in order to situate nulls in specific angles and radiate the main lobe in a desired direction.

The elements of the array antenna are  $d_x$  and  $d_y$  equally spaced. A half wavelength of space between elements is generally used.

The electric field distribution is calculated using equation (8), with the assumption of  $\varphi = 0^\circ$  y  $\theta = 90^\circ$  for H and E plane, respectively. The electric intensity field is calculated with (Leija 2014, Leung 2002):

$$E(\theta, \varphi) = FE(\theta, \varphi)|FA(\theta, \varphi)| \quad (12)$$

where  $FE(\theta, \varphi)$  is the element factor and  $FA(\theta, \varphi)$  is the array factor, calculated with [Frank Gross]:

$$AF = AF_x \cdot AF_y$$

where:

$$AF_x = \sum_{m=1}^M w_m \exp(j(m-1)kd_x \sin \theta \cos \varphi)$$

$$AF_y = \sum_{n=1}^N w_n \exp(j(n-1)kd_y \sin \theta \sin \varphi)$$

here  $w_{mn} = w_m w_n$  is the weighted matrix.

### Least Mean Square algorithm

There are several processes or algorithms for weighting array antennas, focused on minimizing the square error, or mean-square error, at reception. Minimization, in most times, is achieved using gradient-search techniques. One method that has proven very useful, is the least squares algorithm LMS. The LMS algorithm is based on the steepest descent method (Widrow 1967).

The weighted vector is obtained from the direction of the estimated gradient vector. That is:

$$W(j+1) = W(j) + k_s \hat{\nabla}(j) \quad (13)$$

Where:

$W(j)$  is the weight vector before adaptation,

$W(j+1)$  is the weight vector after the adaptation,

$k_s$  is the scalar constant controlling rate of convergence and stability ( $k_s < 0$ ),

$\hat{\nabla}(j)$  is the gradient vector.

A method for obtaining the estimated gradient of the mean-square-error function is to take the gradient of a single time sample of the squared error (Widrow 1967, Gross 2005).

$$\hat{\nabla}(j) = \nabla[\varepsilon^2(j)] = 2\varepsilon(j)\nabla[\varepsilon(j)] \quad (14)$$

Noting that the error is defined as the difference between the desired signal and the reference signal:

$$\varepsilon(k) = d(k) - W^T X(k) \quad (15)$$

Using (15) in (14):

$$[\varepsilon(j)] = \nabla[d(j) - W^T(j)X(j)] = -X(j) \quad (16)$$

Therefore:

$$\hat{\nabla}(j) = -2\varepsilon(j)X(j) \quad (17)$$

The gradient estimate by (17) is unbiased. For a given weight vector  $W(j)$  the expected value of the gradient estimate is:

$$\begin{aligned} E[\hat{\nabla}(j)] &= -2E[\{d(j) - W^T(j)X(j)\}X(j)] \\ [\hat{\nabla}(j)] &= -2[\Phi(x, d) - W^T(j) \Phi(x, x)] \end{aligned} \quad (18)$$

The  $\Phi(x, x)$  matrix is a transversal correlation matrix and autocorrelation of the input signals to the adaptive elements. The matrix column  $\Phi(x, d)$  is a set of cross-correlations between the  $n$  input signals and the desired response signal. Therefore:

$$\nabla E[\varepsilon^2] = 2 \Phi(x, x)W(j) - 2 \Phi(x, d) \quad (19)$$

By comparing (18) and (19), the following relation is obtained:

$$E[\hat{\nabla}(j)] = \nabla E[\varepsilon^2] \quad (20)$$

Thus, given a weight vector the expected value of the estimate is equal to the true value.

Using the gradient estimate equation (13), the value of the iteration rule (17) is:

$$W(j+1) = W(j) - 2k_s \varepsilon(j)X(j) \quad (21)$$

The next weight vector is obtained by adding the vector scaled by the error value to the current weight vector. The LMS algorithm is given by (21) and is usable directly as a formula of weights suitable for digital systems (Widrow 1967).

### Performance of the adaptive LMS algorithm

The desired signal vector  $X_d(j)$  ( $= a_0 s(j)$ ) and the interference signal vector equal to  $X_i(j)$  ( $= a_1 i(j)$ ), are functions of the array steering vector for the desired ( $a_0$ ) and interference ( $a_1$ ) signals, respectively, considered to be equal to (Gross 2005):

$$\hat{a}_0 = \exp [j(N-1) k d \sin (\theta_d)]$$

$$\hat{a}_1 = \exp [j(N - 1) k d \sin (\theta_i)]$$

where  $N$  specifies the number of elements in "x" or "y" directions of the rectangular microstrip array. Likewise, the elements are equally spaced a half wavelength in both directions.

The signal  $s(j)$  is assumed to be the desired received signal, here is considered as:

$$s(j) = \cos(2\pi t(j)/T)$$

The time interval  $T$  is set to 1 ms. And the interference signal is simulated with a vector of random amplitudes:

$$i(k) = randn(1,100)$$

Both signals are nearly orthogonal over the time interval  $T$ .

The total array steering vector is equal to:

$$\hat{x} = \hat{a}_0 + \hat{a}_1$$

With this vector it is possible to calculate the array correlation matrix  $\hat{R}_{xx}(j)$ :

$$\hat{R}_{xx}(j) = \hat{x}(j)\hat{x}^H(j)$$

Where  $()^H$  indicates the Hermitian transpose.

The stability is insured with the following condition:

$$0 < k_s \leq \frac{1}{2\mu}$$

Here,  $\mu$  is the sum of eigenvalues of  $\hat{R}_{xx}$ .

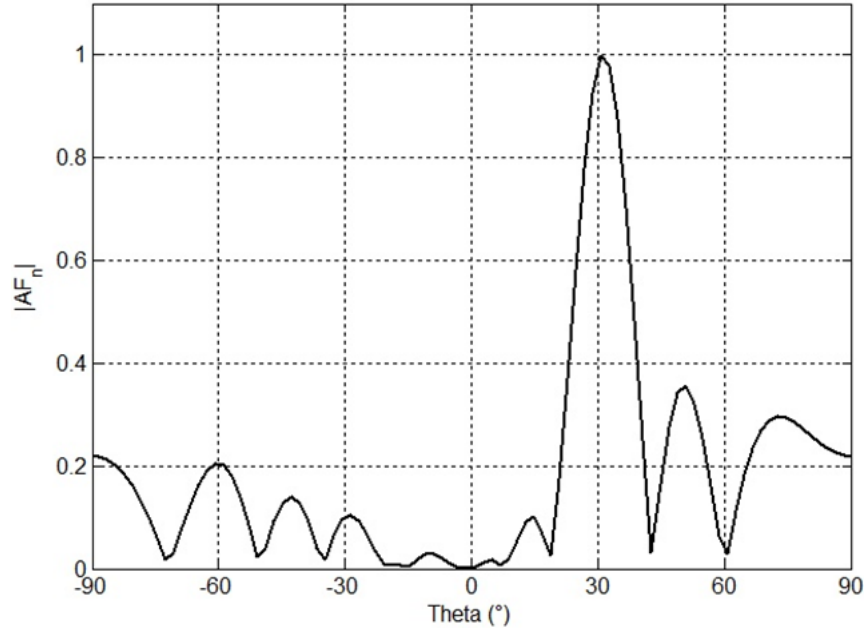
The optimum array weights are calculated applying the Least Mean Square algorithm given by equation (21), where:

$$X_d(j) = a_0 s(j) + a_1 i(j)$$

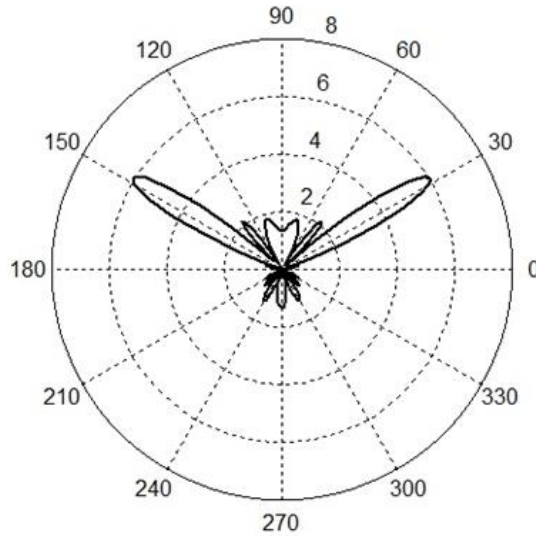
Also, the error signal (Eq. (15)) is calculated. The number of iterations corresponds to the dimension of the signal array  $s(j)$ .

Consider a rectangular microstrip array antenna with  $11 \times 11$  geometry, using patch elements of Figure 1, equally spaced half wavelength in "x" and "y" directions. Assume desired angle for the interest signal of  $\theta_d = 30^\circ$  and an interfering signal with known angle of incidence  $\theta_i = -20^\circ$ . Figure 2 shows the radiation patterns in Polar and Cartesian coordinates.

The complex amplitude of the element on  $m$ th column and  $n$ th row,  $w_{mn}$  is expressed as the product of  $x$ -direction term  $a_m$  and  $y$ -direction term  $a_n$ , i.e.  $w_{mn}$  is merely the consequence of the product  $a_m \cdot a_n$ .



(a)



(b)

Figure 2. Radiation pattern in (a) Cartesian and (b) Polar coordinate system of the weighted LMS antenna array with  $11 \times 11$  configuration.

The calculated weights  $a_m$  and  $a_n$  for the  $11 \times 11$  adaptive array with radiation pattern of Figure 2 are:



$$\begin{aligned} a_m = a_n = & \\ & [1, \\ & -0.15 + 1.10i, \\ & -1.08 + 0.03i, \\ & 0.13 - 1.01i, \\ & 1.15 + 0.022i, \\ & -0.05 + 0.98i, \\ & -1.15 - 0.11i, \\ & -0.02 - 1.02i, \\ & 1.07 + 0.15i, \\ & 0.02 + 1.11i, \\ & -0.99 - 0.11i] \end{aligned}$$

The amplitude and phase signals from each array element are set by the corresponding weights.

## CONCLUSIONS

The design and simulation of microstrip patch antenna array with Least Mean Square (LMS) adaptive algorithm is presented. Also, the way to analytically simulate and evaluate the performance of this type of adaptive array antenna is presented. Initially, the design of inset-line microstrip antenna array is covered. The implementation and simulation of LMS algorithm to the microstrip antenna array is detailed. A  $11 \times 11$  planar array based on microstrip line feed elements was designed for an adaptive beamforming operation system for the frequency of 2.4 GHz. The LMS algorithm is used to generate the complex weights for each element of the array, and steer the beam to  $30^\circ$  direction in H-Plane with a maximum side lobe level of -5dB.

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