SYSTEMIC DESIGN OF A SLIDING MODE BASED MODEL FOR ANALYZING THE PERFORMANCE OF AN ACOUSTIC SENSOR

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ABSTRACT

Currently it is common to use sensors in all aspects of daily life. So, in geophysical processes electronic sensors are required to measure and automate different tasks such as characterization of deep wells, basins, lakes and caves among others. Acoustic sensors are famous because its foundation is mechanical detection, or a sound wave. The acoustic signals are able to be reflected in places which other signals cannot operate either drawbacks or where the liquid moves.

This research aims to develop a systemic mathematical model, representative of the acoustic waves used in acoustic sensors, for analyzing the response to deterministic and non-deterministic variables, also, that assists in the analysis of the damages that have with environmental disturbances, problems which is currently being studied in the worldwide, in order to expand the potential of using acoustic sensors in the global scope.

To achieve the objectives of this research, a systemic and systematic approach methodology is followed, using techniques based on sliders modes to design state feedback control, allowing robustness in the system. Likewise, application results are discussed in the model for optimization. The application of theories and methods, with a systemic and systematic approach enables other form of analysis, interpretation and solve systems problems.

Keywords: Acoustic sensors, mathematical model, sliding mode.

1. INTRODUCTION

In geophysics, the acoustic processes are particularly interesting because described different phenomena in nature, such as the Doppler effects with its relative velocity between two bodies [4]; stochastic entropy [10] with its pernicious effects when the signal exceeds the allowable thresholds [7].

The application of different methodologies in resonance is used to describe the motion of an embryo in a womb or an egg, or fluid levels in an oil well.

Physically, the identification process is based on electronic sensors measuring and building the forms, allowing industrial automatic inspection where man has no access or the human eye cannot see, as the water depth sensing is performed by sonar [8], [9], [6].

To meet the demands of science, multiple types of sensors such as photo-electric, magnetic, inductive, nuclear, acoustic, among others have been created. The acoustic sensor has a great potential still to be exploited in applications such as: content description in deep wells, watersheds, lakes, caverns, applied in places where solutions consider a very rough shape container [6].

In general, the challenges of all acoustic sensors are:

a) The signal type must be consistent with the environment, where the measure movements permit describing the contents,

b) All types of sensors are designed for a range defined without self-adjusting.

An acoustic signal is able to be reflected in areas where other types of signal cannot operate, either by the displacement drawbacks fluid or because they would require excellent reflected signals or with better qualities, i.e., without perturbations. For example, a distance of at least 500 m in aqueous media, operating in power fluid lines with different speeds.

So that, a large distances, the acoustic devices require high quality signals and potency. Instead of it, in this paper presents an acoustic model, taking into account the Doppler signal as a reference adjusting its parameters with an adaptation technique. These adaptive parameters solve the device identification. Offering a high capacity into acoustic sensor with its distance self-regulation. Hence, the device now has a self-regulation distance with adaptation with respect to the reference signal, based on parameters identification system context.

In an event, where the acoustic device needs to be self-adjustable [8], [9] and, [6], the estimation technique needs the adaptive control action. To overcome the current limiting measurable by the device. The control action regulates the acoustic signal level emitted by a piezoelectric, so as to ensure, that the reflected echo signal intensity could be read by the acoustic sensor [1].

The model description, the control system and controller parameter estimation, were the basis for a particular container description, without presupposing known the distance adjusting parameters through the measurement system and control action.

Therefore, the electric model of piezoelectric actuator according to [1] is simplified into the form (1)

$$\begin{aligned} \dot{X}_t &= AX_t + Bw_t \\ Y_t &= CX_t + Dw_t \end{aligned} \tag{1}$$

Where, the matrix order corresponds to differential first order equation.

Theorem 1. *Let the model considered in [1] in state space has the form (1). In agreement to [3] and (1), the recursive form is (2).*

$$\dot{Y}_t = \boldsymbol{G}Y_t + \boldsymbol{H}V_t \tag{2}$$

Where: $\boldsymbol{G}$, $\boldsymbol{H}$ *are matrices bounded with* $\boldsymbol{G} \in \mathbb{R}^{[\boldsymbol{n}\times\boldsymbol{n}]}_{[\boldsymbol{0},\boldsymbol{1}[}$ *and,* $\boldsymbol{H} = f(A, C, B, D)$, $V_t \in N\{\mu, \sigma^2 < \infty\}$

Proof *(See Annex).*

2. CONTROL LAW

Theorem 2. *The control law system with respect to (2) has the form (3)*

$$V_t^{\,*} = H^{+}(E_t^{\,*} - GY_t) \tag{3}$$

With H^{+} *the pseudo-inverse matrix* H*, the innovation process* E_t *considered in (2), and,* G *as a matrix having the form* $G \in \mathbb{R}^{[n\times n]}_{[0,1[}$.

Proof *(See Annex).*

The piezoelectric main problem corresponds to parameter distance description [4], [1]. It is solved estimating this gain with a lower uncertainty in almost all points that make up its surface.

3. PARAMETERS ESTIMATION

Applying the control law (3) into the model described in (2), converge to reference system (Y_t) only if it is known the matrix $\boldsymbol{G}$[1], [3]. Unfortunately, the reference system viewed as a black-box scheme [3]; the matrix gain $\boldsymbol{G}$is unknown, because correspond to the internal system description. Consequently, the estimation process is required describing the internal matrix gain through the time process [3], [5], [2].

Theorem 3. *Let the recursive model (2), with answer corresponds to the reference output system, the stochastic matrix estimation is (4)*

$$\widehat{\boldsymbol{G}}_{\boldsymbol{t}} = \boldsymbol{P}_{\boldsymbol{t}}\boldsymbol{Q}_{\boldsymbol{t}} \tag{4}$$

With $\boldsymbol{P}_{\boldsymbol{t}}$, $\boldsymbol{Q}_{\boldsymbol{t}}$ *covariance and variance matrices, respectively with respect to (2).*

Proof *(See Annex).*

Theorem 4.*The recursive form of (4) in discrete manner with stationary conditions is (5).*

$$\widehat{G}_t = \alpha_t \widehat{\boldsymbol{G}}_{t-1} + \beta_t \tag{5}$$

Proof *(See Annex)*.

A piezoelectric device developed as a mathematical model viewed as (1), considering the black-box properties, only is observed input and output signals without knowing exactly the internally system operations. The control law not affects the reference system but the model through the parameters is estimated in the probability sense affecting the model converging to the reference piezoelectric device answer. All results are described into the real numbers ($\mathbb{R}$), specifically over the hypothetical line that describes the container form.

The piezoelectric block diagram using the control action with adaptation into model system, and it is shown in figure 1.

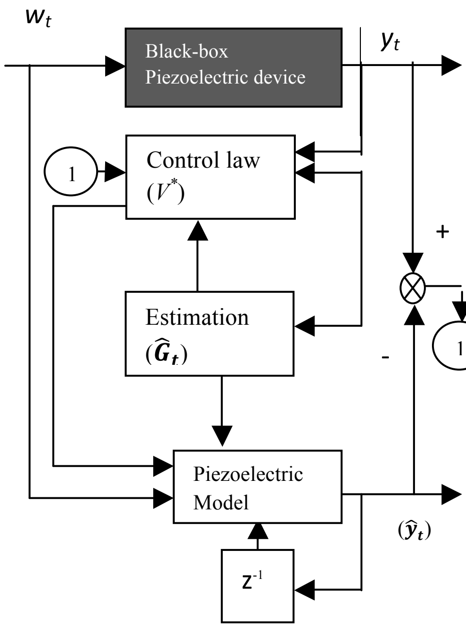

Figure 1. Piezoelectric system viewed as a control block diagram with parameters adjusted dynamically

4. SIMULATION

The piezoelectric signal (y_t) with stochastic properties tracking by the model output answer ($\hat{y}_t$) with parameters estimation affecting the control law action, both considered in (2) is illustrated in figure 2.

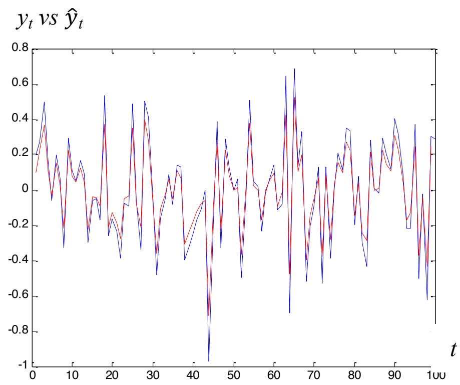

Figure 2. Piezoelectric temporal signal (blue) and its tracking (red).

The adaptation scheme viewed in figure 1, was applied in the control law and into the model, observing that the answer is very narrow with respect to the real piezoelectric results.

In figure 3, is observed the innovation process.

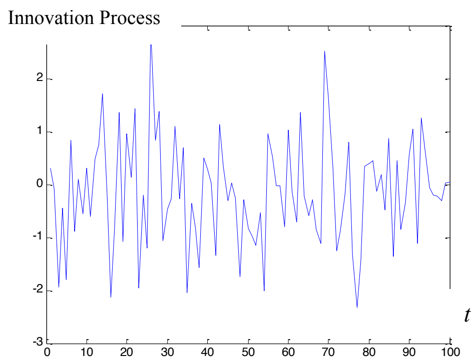

Figure 3. Innovation process through the time t

The convergence rate generated between the reference signal and the tracking output model is measured in decibels based on stochastic entropy and has the form $H_t = -20[e_t ln(e_t) - (1/20)H_{t-1}]$, with$e_t := y_t - \hat{y}_t$and the results is viewed in figure 4.

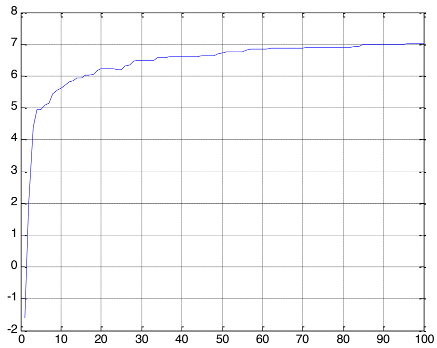

Figure 4. Stochastic entropy with respect to difference between the tracking and reference signal

CONCLUSIONS

This paper presented a model considered into the references as a piezoelectric device description. The control system over the model was required with a matrix gain parameters, having and adaptive movements and helps into the tracking operations. The theoretical results affect the dynamical model properties and the control action. The simulation tracking was acceptable in probability sense, with convergence rate measured in stochastic description manner, in decibels. The piezoelectric device output has an evolution with innovation signal. The tracking permits a great convergence with stationary conditions affecting the control action over the model in positive form, minimizing the convergence error near to piezoelectric answer with random occurrence. The control law depends on the internal parameters, with adjustable gains estimation. The simulation results describe the results proposal, with a convergence level bounded in its movements as shown in the entropy figure.

ANNEX

Proof (Theorem 1).

Let the model (1)*, with the first derivate described in* (4).

$$\dot{Y}_t = C\dot{X}_t + D\dot{w}_t \tag{4}$$

Substituting in (4) *to* $\dot{X}_t$ *de* (1)*, has* (5)

$$\dot{Y}_t = CAX_t + CBw_t + D\dot{w}_t \tag{5}$$

The internal state X_t *in agreement to* (1) *is described in* (6) *with respect to observable signal.*

$$X_t = C^{+}Y_t - C^{+}Dw_t \tag{6}$$

Considering (6) *in* (5)*, has* (7)

$$\dot{Y}_t = CAC^{+}Y_t - CAC^{+}Dw_t + CBw_t + D\dot{w}_t \tag{7}$$

In symbolic form (7) *is described in* (8) *with* $\boldsymbol{G} \coloneqq CAC^{+}$, $\boldsymbol{H} \coloneqq [(-CAC^{+} + CB) \quad D]$, *and* $V_t \coloneqq [w_t \quad \dot{w}_t]^T$.

$$\dot{Y}_t = \boldsymbol{G}Y_t + \boldsymbol{H}V_t \tag{8}$$

Corresponding to (2).

Proof (Theorem 2). *Let the system* (2) *accomplish with* (9)

$$M_t\dot{M}_t^T < 0 \tag{9}$$

Where the trajectory region with respect to the gain matrixM_t, *is described in* (10)

$$\dot{M}_t = -F_t \tag{10}$$

With F_t, *a continuous function bounded by intervals with uniform measure t, accomplishing with the innovation process* (11)

$$\dot{\hat{Y}}_t = \dot{Y}_t + F_t \tag{11}$$

Considering to (2) *in* (11) *has* (12)

$$-F_t = GY_t + HV_t - \hat{Y}_t \tag{12}$$

The difference between $\dot{\hat{Y}}_t and F_t$*in* (11) *and agreement with* (12) *has* (13).

$$\left(F(t) - \hat{Y}(t)\right) = -GY_t - HV_t \tag{13}$$

The system inovation process (2) *according to* (11)*, it is described in* (14)

$$E_t = \dot{\hat{Y}}_t - F_t \tag{14}$$

(14) *in* (13) *and* V_t*, is develop in* (15)

$$V_t{}^{*} = H^{+}(E_t{}^{*} - GY_t) \tag{15}$$

As a control law, described in (3).

Proof (Theorem 3).

Let $Z_t^T = f(Yt)$, *such as applying the second probability moment* (2) *with respect to* Z_t^T, *has* (16)

$$\boldsymbol{E}\{\dot{Y}_t Z_t^T\} = \boldsymbol{GE}\{Y_t Z_t^T\} + \boldsymbol{HE}\{V_t Z_t^T\} \tag{16}$$

The mathematical operator properties in agreement to [3], [5] and, [2], the estimation is based on (16) has the form (17)

$$\widehat{\boldsymbol{G}}_{\boldsymbol{t}} = \left(\boldsymbol{E}\left\{\dot{Y}_t Z_t^T\right\} - \boldsymbol{H}\boldsymbol{E}\{V_t Z_t^T\}\right)\left(\boldsymbol{E}\{Y_t Z_t^T\}^{+}\right) \tag{17}$$

Now, defining (17) with respect to (18) to $\mathrm{P_t}, \mathrm{Q_t}$ in (18)

$$\boldsymbol{P}_t \coloneqq \boldsymbol{E}\left\{\dot{Y}_t Z_t^T\right\} - \boldsymbol{H}\boldsymbol{E}\{V_t Z_t^T\} y \boldsymbol{Q}_t \coloneqq \boldsymbol{E}\{Y_t Z_t^T\}^{+} \tag{18}$$

And (18) in (17), the estimation matrix $\widehat{\mathbf{G}}_{\mathbf{t}}$ has the form (19)

$$\widehat{\boldsymbol{G}}_{\boldsymbol{t}} = \boldsymbol{P}_{\boldsymbol{t}}\boldsymbol{Q}_{\boldsymbol{t}} \tag{19}$$

Viewed in (4).

Proof (Theorem4). Let (5) has the components in agreement to (18), with stationary conditions $\mathbf{P_t}$is described considering [3], [5], [2] in recursive form (20).

$$\boldsymbol{P}_t = \frac{1}{t^2}\sum_{i=1}^{t}\left(\dot{Y}_i Z_i^T\right) + \frac{\boldsymbol{H}}{t^2}\sum_{i=1}^{t}\left(V_t Z_t^T\right) \tag{20}$$

With a delay, $\boldsymbol{P}_{t-1}$ *is described in* (21)

$$\boldsymbol{P}_{t-1} = \frac{1}{(t-1)^2}\sum_{i=1}^{t-1}\left(\dot{Y}_i Z_i^T\right) + \frac{\boldsymbol{H}}{(t-1)^2}\sum_{i=1}^{t-1}\left(V_t Z_t^T\right) \tag{21}$$

(21) in (20) has (22)

$$\boldsymbol{P}_t = \left(\frac{1}{t^2}\right)\left[\dot{Y}_t Z_t^T + \boldsymbol{H}V_t Z_t^T + (t-1)^2\, \boldsymbol{P}_{t-1}\right] \tag{22}$$

Considering that (19) delayed has the form (23)

$$\boldsymbol{P}_{t-1} = \widehat{\boldsymbol{G}}_{t-1}\boldsymbol{Q}_{t-1}^{+} \tag{23}$$

And (23) in (22), has (24)

$$\boldsymbol{P}_t = \left(\frac{1}{t^2}\right)\left[\dot{Y}_t Z_t^T + \boldsymbol{H}V_t Z_t^T + (t-1)^2\widehat{\boldsymbol{G}}_{t-1}\boldsymbol{Q}_{t-1}^{+}\right] \tag{24}$$

(24) in (19), has (25)

$$\widehat{\boldsymbol{G}}_{\boldsymbol{t}} = \left(\frac{1}{t^2}\right)\left[\dot{Y}_t Z_t^T + \boldsymbol{H}V_t Z_t^T + (t-1)^2\widehat{\boldsymbol{G}}_{t-1}\boldsymbol{Q}_{t-1}^{+}\right]\boldsymbol{Q}_{\boldsymbol{t}} \tag{25}$$

Minimizing (25) has (26)

$$\widehat{G}_t = \left(\frac{\dot{Y}_t Z_t^T \boldsymbol{Q}_t}{t^2}\right) + \left(\frac{\boldsymbol{H}V_t Z_t^T \boldsymbol{Q}_t}{t^2}\right) + \left(\frac{(t-1)^2\widehat{\boldsymbol{G}}_{t-1}\boldsymbol{Q}_{t-1}^{+}\boldsymbol{Q}_t}{t^2}\right) \tag{26}$$

(26) Symbolically is described in (27)

$$\hat{G}_t = \alpha_t \widehat{\boldsymbol{G}}_{t-1} + \beta_t \tag{27}$$

With α_t*and* β_t *as* (28)

$$\alpha_t = \frac{(t-1)^2 \widehat{\boldsymbol{G}}_{t-1} \boldsymbol{Q}_{t-1}^{+} \boldsymbol{Q}_t}{t^2}$$

$$\beta_t = \left(\frac{\dot{Y}_t Z_t^T Q_t}{t^2}\right) + \left(\frac{HV_t Z_t^T Q_t}{t^2}\right).$$

And (27) *is viewed in* (5).

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