RELATIONAL SCIENCE: A SYNTHESIS

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1. Introduction

This paper proposes a synthesis of two avenues of development of relational complexity theory that have followed the work of Robert Rosen (1934-1998). These avenues are: (1) further development of Rosen's causal entailment in category theory (Rosen 1978 Rosen 1985 Rosen 1991 Rosen 1999 Louie 2009); and (2) contextual entailment based on Rosen's modeling relation (Rosen 1985 Rosen 1991 Rosen 1999 Kineman 2007 Kineman 2008 Kineman Banathy and Rosen 2007 Kineman and Kumar 2007). These two tracks represent different theory structures that have not been fully integrated to date. Category theory describes causes in terms of entailments expressed as mappings between sets of a domain and co-domain. Modeling relations describe a complementarity between descriptive and, as argued here, prescriptive potentials of a system and their natural realizations; mediated by information relations. The synthesis presented here combines these two theory tracks to bring their mathematical and graphical systems of analysis into correspondence with each other and with a natural interpretation of causality. Such a synthesis requires asking if the current causal mapping algebra is sufficiently comprehensive to describe natural modeling relations, or alternatively, if the application of modeling relations as a fundamental analysis of nature requires additional algebraic elements.

Relational theory currently struggles to express its ideas in a familiar but out-dated language that is restricted by mechanistic metaphors. The field can only be united by a new mathematical language that is complete in its treatment of causality, and rooted in natural meanings. Louie's book, "More Than Life Itself" (Louie 2009) advanced many of the mathematical concepts that were borrowed from category theory as Rosen and his followers applied it to relational biology. Louie described the basic application of category theory to relational biology in the style of pure mathematics while injecting useful philosophical references that help make many meanings clear. The book is destined to stand as a milestone in the development of this new language of relations. Nevertheless, in both Rosen's and Louie's treatments, modeling relations remained on the periphery, serving to describe the way science works and to provide tests for, or indicators of, complexity, but not to analyze that complexity. The characteristics of modeling relations have been used as an argument for expansion beyond mechanistic analysis but they have not been brought fully into the formalism of complex analysis. The result is that the current relational algebra continues to present a 'bottom-up' legacy by implicating complexity only from the organization of efficient entailments, rather than incorporating the ‘top-down’ causalities of contextual entailments. Defining causal maps as elements of modeling relations provides the needed framework for describing and analyzing nature far more deeply than has been possible otherwise, in terms of fundamentally complex systems that can be reduced to act as mechanisms or organized into more self-entailed complexities such as life.

This work continues the development of relational biology and its characteristic association with the study of causality. Specifically, it re-interprets and integrates the four Aristotelean causalities (or modes of natural explanation) – material, efficient, formal, and final – using the framework of modeling relations. To a large degree this work follows directly on Louie’s call for an age of synthesis in which “modes of synthesis would include the entailment of existence: immanent causation.” The relationship between actual (realized) and contextual (potential) aspects of nature described here is very much entailment of existence. We are learning that complexity is the coincidence of four causes that were introduced by Aristotle but inadequately understood and only partly adopted since then. The mathematical integration of these causes into a recursive, hierarchical framework, as proposed here, provides a new and deeper understanding of causality.

That integrated causality is presented as a relational holon after the popular term coined by Arthur Koestler (Koestler 1967 1978). A holon is a unit of analysis (attributed to nature), that is "simultaneously part and whole.” Here it is developed as a composition of logically inverse categories that exchange the roles of structure and function (used in their natural sense, stated most briefly by Rosen (1971) as “what it is” and “what it does”, respectively). To describe modeling relations explicitly in this way, we need to make a clear distinction between realized and contextual domains, and accordingly to introduce symbols to directly represent contextual maps. While category theory is flexible enough to describe contextual maps and thus to allow this distinction, formulations of relational theory including Rosen’s and Louie’s have not explicitly done so, leaving the contextual side of the modeling relation vague. Context is therefore a central theme of this synthesis.

Furthermore, category theory generally describes mathematical transitions between sets of elements (of broad type); whereas relational theory does not presume that nature contains such elements prior to the abstraction of properties through various system interactions. As Rosen and others (e.g., Bateson 1979) have argued, properties or states are abstracted from an otherwise unattributed reality in (and by) natural contexts. Both Rosen and Louie emphasized that the notion of a material state is no less abstract than a system’s formal aspects. Rosen wrote “There is nothing more abstract than a number” (Rosen 1991). In that case, relational theory describes the process of attributing a natural system; and accordingly, the abstracted states should not be treated as pre-existing. The new mathematical conventions that are introduced here change the meaning of a relational mapping from a mapping between elements, to an abstraction of elements. The resulting theory structure is more robust, being applicable to phenomena in many fields, and meeting important epistemological criteria that justify the expanded view.

From the integral perspective of this synthesis, when we say that complex living systems have “internal models” (both Rosen and Louie), it follows without equivocation that natural systems somehow produce such models and have modeling relations just as we do when we conduct science (or any other thought derived from and applied to nature). In other words, there must be natural analogs to epistemological modeling relations and their characteristics. Rosen described the condition of internalized entailment as ‘impredicativity’; the case where internal causality is self-defined and thus not predicated on causes in the environment. Hence, the question that must be asked is if modeling relations apply generally in nature: Is their validity restricted to the description of science or the world of organisms, or do they have broader meaning for nature as a whole? The radical proposition that seems inescapable here is that the principle of modeling relations is general to all systems; that living systems are a special organization of such relations (described by Rosen as “M-R systems”), and that mechanisms are a reduction of that complexity (Kineman, 2007). In this case Rosen’s belief that biology can reveal more fundamental principles of nature than any other field, may be true. Specifically, the adaptive niche concept may exemplify most clearly and generally the contextual domain by which science can be expanded to include higher order causes that account for complexity. Modeling relations bear a striking resemblance to niche relations in ecological theory (Kineman 2009b). Hutchinson’s (1953) n-dimensional niche specifies conditions for existence of an organism. Context can thus be defined as a domain in which the conditions for existence of any phenomena (irrespective of subsequent adaptation) are specified by means of modeling relations.

If category theory is truly to be “the general theory of modeling relations” as Louie claimed (Louie 2009, pg. 329), it must be able to describe them as whole causal units in nature, comprising explicit representations of each of the four causes and their mappings. At present it does this only partially. Specifically, formal and final causes were not explicitly mapped by Rosen or Louie except in modeling relations. Otherwise, they remained hidden from the diagrams of categorical mappings, suggesting their treatment as emergent properties of efficient and material entailment. Only a weak link between the full set of causes could thus be implied by the relational algebra. The first task of this synthesis should therefore be to clarify and relate the four causes in a clear analytical framework.

The specific issues that are addressed here to achieve the proposed synthesis are:

1. While there are criteria for defining complex vs. simple systems, how we get from one to the other, or how nature does so, is not currently clear. The present theory provides indicators for complexity, but only a weak explanation of its origin, with no analytic bridge between these types of systems (Louie 2009, pg. 227). Closure of efficient entailment, the cited basis for complexity, cannot be both impossible as a purely efficient specification and definitional as a criterion for complexity at the same time. Therefore, a hierarchical circular entailment of efficient causes (Louie 2009, pg. 126-127) presents an unresolved paradox, suggesting that the theory must take a broader view.

2. In complex systems such as ecosystems, functions combine and have multiple pathways that may have unknown causes or system dependencies that do not translate into general causality. The rule that “a mapping uniquely determines its domain and co-domain” (Louie 2009, pg. 122), which establishes a clear mathematical syntax for mappings, should not be interpreted in a way that prevents showing dualistic maps at any causal level.
3. Rosen introduced the idea that an ‘inertial thing’ can be a ‘gravitational thing’ (an insight he attributed to Schrödinger); meaning that a caused structure can itself become causal (Rosen 1999). This principle is what allows the composition of causal mappings, where an efficient cause (solid headed arrow) can be drawn from an object that was itself the result of a previous mapping (hollow-headed arrow). In current practice, therefore, “There is nothing in category theory that mandates an absolute distinction between sets and mappings” (Louie 2009, pg. 120). While we may accept that a mapping describes an entailment of sets like a natural function analogously causes change in terms of system structure, something more is needed to explain how a structure causes a function. We need the theory to tell us how this magic occurs; how sets and mappings are defined in different natural domains so that inertial and gravitational objects can be related.
4. The causal hierarchy needs further clarification. Historically, the hierarchical relation between efficient and formal cause has been unclear. Relational theory suggests the natural hierarchy: material—efficient—formal—final, placing formal cause above efficient cause and identifying it with systemic properties of the whole. That view is supported by Rosen’s work, but there has been an equivocation since Aristotle’s time, in which efficient and formal causes are sometimes exchanged in the hierarchy and formal cause is described in the sense of an efficient shape, like a mold. The question can be put this way: Do processes create design or do they realize design? Louie represented the causes with both possibilities (Louie 2009, pg. 113), but it is critical for establishing a consistent theory and method of analysis to define the causes such that they have a unique hierarchical order.
5. Finality (final cause) must also be clarified further. Identifying finality with the output of a mapping (Louie 2009, pg. 113) overlooks the explanation of how or why the output influences the prior entailments and thus constitutes final cause. As a mere end, it is not clear how it is non-trivially causal.

This paper will show that re-interpreting and relating the fundamental causes as a recursive hierarchy (the relational holon) and equating that hierarchy with an analytical form of the modeling relation, resolves these issues and establishes context as a natural domain of existence with its own causes. The argument proceeds by examining the generality of modeling relations.

2. Modeling Relations and Contextual Dualism

Rosen used modeling relations **(Fig. 1)** to describe science ideally as a commuting relation between natural systems and formal (mathematical) systems (Rosen 1991), labeling

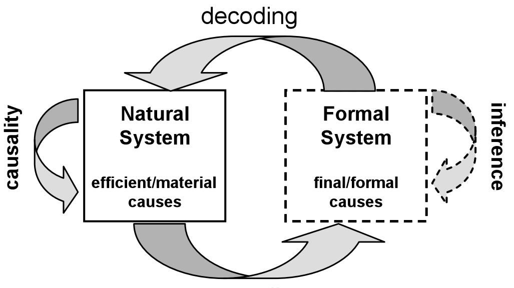

Figure 1: The Modeling Relation

the contextual side (right) as inference. However, they can be more generally understood as relations between locally real and contextually potential aspects of natural systems (Kineman, 2007; Kineman 2008; Louie 2009, pg. 91). A 'formal system' in mathematics provides a cognitive scientific context for representing external systems, but itself has locally realized properties: in the brain, in a computer model, and other examples. As explained here, contextual systems are, in all cases (cognitive and natural), systems of final and formal cause. Context is that aspect of a natural system that provides the conditions for existence of another system; attributing the system as a result of interaction or observation (making no distinction at this stage between conscious, living, or physical systems). Context (synonymous with ambiance in Rosen's usage) is necessarily a domain of non-localized mirror images of realized local events, and accordingly the modeling relation describes a necessity in nature where these two aspects of reality mutually establish each other. Non-locality refers to contextual potentials: unrealized constraints in a contextual system that establish the potential existence (formal cause) of another system. It is very much like an ecological niche factor space in which a system can be identified, aside from its actual occurrences, by its ability to exist in various factors that are realized independently. The niche-space thus identifies potential existence in factors other than time and location (which are properties of realization).[1]

Modeling relations are thus relations between measurable properties and inferable potentials of nature. We can call the localized aspects real or realized, and we can call the non-localized aspects contextual or contextualized. The term 'whole' must then be reserved for their combination. The realized domain is where events act or can be known in terms of local properties of a system. We are generally familiar with the localized domain of realization as 'the world', but more technically, the space-time interactive existence that also provides the coordinates for sensory perception. The non-localized domain is one of identities, characterized by potential existences, which we can legitimately call natural models (quite analogously with the concept of epistemological models) by which objects in space-time have identity in terms of the conditions under which they can exist. These opposite domains are not separate systems, but dualistic aspects of a whole (as with the concepts of insides and outsides) that define and establish each other and the whole, recursively through modeling relations, as shown in **Fig. 1.** Hence the domain of perception and interaction (the natural, realized world or system) is itself established by events that are a realization (measurable effect) of contextual potentials. The complement of this effect is specification, by events, of a region of contextual potential (a model) that identifies and associates those events as a system. Continuity of material existence requires this relation to exist in a stable form, and yet its intrinsic instability is what allows complex possibilities. Nature is thus complex because its modeling relations establish and attribute contextual recursive representations that both reflect and differ from their realizations. Thus the implied commutativity in **Fig. 1** must be considered partial in this sense, or applying to selected aspects of a system.

Potentials for the possible existence of a system occur as a set of conditions within a contextual system, which can be described as a factor or variable space with any number

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[1] The concept of non-localized potential can also be described by the physical science concept of a 'phase space', but one that excludes space and time coordinates.

of non-spatiotemporal dimensions. In ecology this set of conditions is the adaptive niche. The range of possible conditions is a model for existence established by prior existence and recursively realized as local events in space and time. Such potentials or models are causally inverted system images that can act as attractors of functions that realize them. The relation between measurable properties of a system (local properties in space and time), and their contextual existence as such potentials (suitabilities mathematically similar to probability densities), can be associated with the concept of information (one system 'formed in' another). Human experience of information would correspond with use of such natural relation.

In the philosophy of science, formal mathematical models and the realized systems they represent in nature are differentiated and related in just the way described in **Fig. 1**; although we often forget that scientific models are not the reality itself. Ideally in science, we specify models as symbolic systems that commute with and thus describe natural systems, thus establishing an epistemological modeling relation that describes nature via information relations, specifically encoding and decoding.[2] As we see that much of nature itself behaves like modeling relations, it becomes clear that the very practice of science recapitulates the entailment of nature; which is one explanation for why science is possible and why mathematical descriptions work (the "surprising efficacy of mathematics" cited in Louie 2009, pg. 99). However, science can describe different things; it can be about discovering unique phenomena to improve our technical capacity to make changes, or about analyzing whole systems to improve our systemic interactions and fitness in nature. The difference is very important with regard to methodology. If we want to invent new technology, incomplete knowledge, even understanding of a single cause, is often a sufficient (even revolutionary) beginning; but if we wish to understand an existing system in order to deal with its complexities, we must strive to understand it completely, including all modes of entailment that inhabit the two domains mentioned.

Clarifying further, the efficient/material domain is that of natural realization, which is local and measurable, and the final/formal domain is that of contextual potential, which is a suitability for existence that is non-locally specified in variables prior to their independent local realizations. Such potentials are knowable only by inference. In simpler language, the context is the 'background' of conditions that can be described as a natural model for the actual, measurable, 'foreground' of events. The key to relational theory is to objectify both domains, and especially their relation. Mechanistic science tries to objectify only the events and their fixed laws, but that approach fails when complexity dominates. As currently applied, category theory objectifies relations in terms of objects and their morphisms, and it represents complexity by allowing these terms to change places; but it does not yet explain why or how that role reversal takes place.

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[2] Rosen distinguished between true models in a scientific sense and simulations, in that true models must reflect natural entailment. In that case, given the syntheses here, a true model is one that considers realized and contextual causes.

2.1. Generality of modeling Relations

The idea that contextual potentials and realized events cause each other is perhaps most obvious in ecology. An organism's niche has active properties: It can be said to attract realized systems (an organism) that are in some way pre-defined. In the absence of such realizations, over a sufficient length of time, context can be said to realize that potential (for the organism, or a similar one) through evolution. In fact it is necessary in evolution theory to give environment a causal role, which is selection by context from the top-down; whereas in physical science it is more common to think that everything is caused from the bottom-up, by processes. That difference in the direction of causality does not exist in the mechanistic point of view, in which contextual cause must be considered metaphorical or reducible. Relational theory combines both effects as true and irreducible causalities to provide a more complete analysis. We are thus obligated to consider not only the causality of environments and ecosystems with respect to organisms, but also the causality of organisms with respect to their environment and niche potentials (e.g., Odling-Smee Laland and Feldman 2003). The organism-environment system is a two-way contextual (modeling) relation, giving us a clue to more fundamental existence of this relation in nature.

Generalizing from these ecological principles of complexity (where nature's full causality is seen) to the rest of nature, we can see evidence of the same fundamental relation between physical events or systems and their contextual potentials in cases where the origin and definition of the system are part of the analysis. That evidence occurs in many phenomena, for example in quantum, relativistic, thermodynamic, and dissipative systems. Understandably contextual relations dominate in more highly organized (and variously defined) systems, including ecological, social, political, economic, and other cognitive systems. But, there is no sharp dividing line between the complex and the non-complex, no 'threshold' that can be defined theoretically, because mechanism is a reduction of complexity toward a completely defined (a fully commuting) modeling relation, which is a condition that can never be fully attained. We build machines that perform their mechanistic design purpose well enough and we quantify mechanisms in nature well enough for specific predictive purposes, while all of these systems retain their natural complexity in a deeper analysis.

We must, therefore, consider the relation between such realized and potential aspects of nature to be fundamental to anything that can be said to exist. The difference between mechanistic and relational analysis is the degree to which that relation is accounted for. The restrictions or constructions on that basic complex relation determine the mechanistic qualities of a system, and special entailments that establish living systems. In this sense, mechanism is never discarded; it is transcended and included in a recursively complex relational view. Louie wrote: "Any natural system has many different models that do not contain impredicative structures of inferential entailment, whence they are simulable" (Louie 2009, pg. 231). Living systems, especially, represent a case that might be called 'super-complex' systems, in that they establish a special kind of organization that takes great advantage of both mechanical and contextual entailments.

2.2. Contextual Relations and Holarchy

Encoding and decoding are the relations described by Rosen that establish modeling relations. Since they are obviously information relations, it should be clear that information is an inescapable concept in relational theory and its explanation of nature, as Gregory Bateson, for example, insisted (Kineman and Kumar 2007). However, as Rosen pointed out, information (encoding and decoding) between related systems must transcend both the natural system and its model in order to connect them; and these relations must therefore be part of a "larger" more general context of the whole modeling relation itself, implying an endless hierarchy. One of Rosen's definitions for a mechanism was a truncated hierarchy of this sort, because a largest descriptive context is necessarily a syntactically complete system, and thus defines a mechanism.

Fig. 2 shows part of the infinite hierarchy of modeling relations (the contextual system is indicated by a dashed border, and the hierarchy is meant to continue indefinitely). Rosen's criterion for a complex system, that it not have a complete description, is thus met where no context constitutes such a largest system capable of describing it without further inputs. A mechanistic reduction or truncation of the hierarchy indicates that a natural system is sufficiently identified with its environment causally such that a common set of laws can describe both. Complex systems thus exist between the theoretical extremes of isolation and connection; they are independently identified sub-systems and interconnected components of greater contexts. This principle of relative causal isolation and connection will be an important key to explaining how relational mappings compose in an analysis of nature.

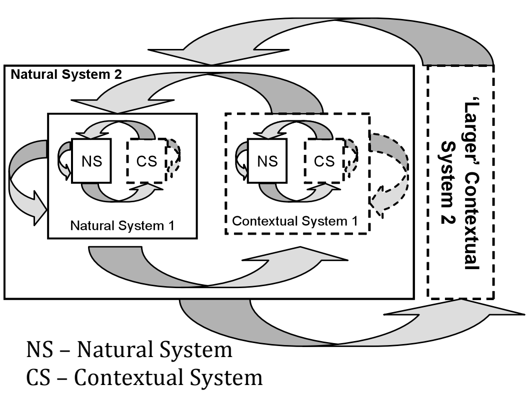


Figure 2: Implicit Hierarchy of Modeling Relations.

If modeling relations are already relations with context, then the infinite hierarchy of contexts (**Fig. 2)** can be equivalently represented as a network of contextual relations, as shown in **Fig. 3.** As such, each system provides a context for other systems, thus defining a holarchy that may also comprise multiple alternative hierarchies. Any natural system is implicitly both a component of other systems and a composite of systems; and may be analyzed without limit in terms of deeper internal causes and/or formative external causes, thus making analysis relative to multiple perspectives**.** Such networks are holarchical because they can represent direct relations that summarize a decomposable chain of relations at any hierarchical level (as in the example of "Natural System 2" in the diagram). In this way, it is possible to keep the implicit infinite regress of larger or smaller systems within the definitions of nature and subject to analysis. This representation then allows the association to be made (later) with relational mapping algebra.

It is important in any analysis to be able to focus on proximal causes at the expense of distal and presumably less significant ones. As described in the preceding paragraph, deep relations can be summarized by more general holons, choosing the level of specificity desired. For example, in **Fig. 3** one can summarize System 1 with System 2 in an equivalent manner to **Fig. 2**, by showing its contextual relation to Natural System 1 (thus implicitly including System 1 relations). While it may seem impossible, then, to know if more distal relations (with respect to one's analysis) might end up driving the system (as in the supposed "butterfly effect"), the holarchical assumption represented here means that even a very long chain of relations, if found empirically to be important, can be summarized as a proximal one (with respect to the systems of interest); albeit, with uncertainties inherent in that chain.

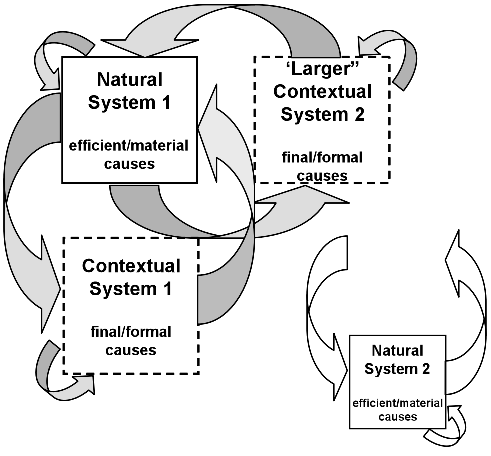


Figure 3: Equivalent Holarchy of Contextual relations.

These principles apply to living and non-living systems alike, at their most fundamental level. We may not see, however, the effects of complexity in commonly experienced events. In this theory information (encoding and decoding) about the natural world is being shared, not just by cognitive organisms but by all systems with intersecting histories and active interactions. In the absence of causal system boundaries, a common reference frame is established by events. Relational theory, while initially motivated by the exploration of living systems, arrives at a view of all natural phenomena as being fundamentally complex, but reduced by interaction.

The mechanistic view hides the complex origin of systems, assuming they are already reduced to a common ccontext. To consider origins in any terms requires consideration of the 'higher' contextual causes. That consideration need not reach outside of nature if the causal aspects of context can be unified with the apparent causality of realized events. In attempting to find that unity relational analysis is first qualitative, describing system organization, and second quantitative; because quantity is predicated on the contextual identification or origin of a system or event; in other words, its identity. The identity of a system prior to its realization is thus what ensures complexity of all systems at their origin.

2.3. Causality Redefined: The Relational Holon

From the perspective of relational theory our formal view of causality has been incomplete, but scientific integration of Aristotle's causal levels has proven to be elusive. In the work of both Rosen and Louie, the theories of causality represented by modeling relations and relational mappings (based on category theory) were not fully integrated; but by resolving differences in how causality is treated in these two approaches, a logically consistent integration is possible. The first step in this process is to fully incorporate contextual relations into the causal explanation of phenomena. To accomplish that task, we need to re-interpret Aristotle's four causes as a recursive hierarchy of discrete, irreducible causes that act between levels and that relate ontological and epistemological domains. The result is a more explicit definition of a modeling relation as a natural and analytical unit (**Fig. 4**), which, for need of an identifying term, can be called a holon.

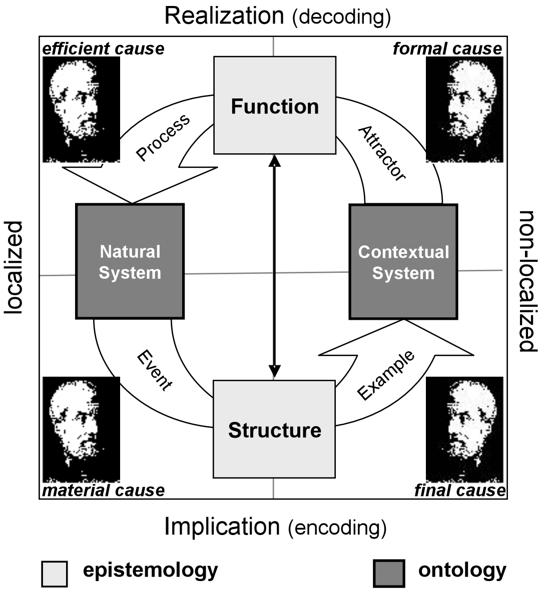
This definition of the holon gives much more detail about modeling relations, associating the quadrants of the diagram with Aristotle's four causes (as re-interpreted here) and revealing that modeling relations translate the ontological concepts of natural system and contextual system (dark boxes on the horizontal axis) into the epistemological concepts of natural structure and function (light boxes on the vertical axis), giving these terms much needed definitions as natural forms of encoding and decoding, respectively (Kineman Banathy and Rosen 2007). Structure can be understood as abstraction from nature of a 'code' in the form of a measurable pattern, which becomes the exemplary basis for a natural, contextual model. Function can be understood as the expression of a code abstracted from a contextual model as an attractive potential that drives processes via functions (the term 'attractor' is meant here in an active, causal sense). The four quadrants in the diagram thus correspond uniquely and recursively to each of the four causes.

The left and right sides of the holon employ opposite logic (complementary categories). The left side is deductive and the right side is inductive. Just as science requires both sides, so does nature, and in this sense science is a natural activity. The endless possibilities for relating these complementary domains are recursive and composable. The holon represents a unit of natural holism; the antithesis of the "fundamental particle" that classical science searched for but did not find in the material domain. The logical framework of the holon suggests that all phenomena can be described in terms of such units, through their causal compositions and decompositions into internal or external relations and entailments.

This view allows a clear definition of terms related to complexity. The term closure refers to a mutual exchange of causes between two or more holons in one or more quadrants. The phrase hierarchy of causes (branching or circular) refers to the relationship between the causal levels. These two concepts are co-dependent because, as we will see later, causal closure necessarily involves a circular path through the causal hierarchy, but the terms have distinct meaning in the analysis. The circular hierarchy describes system identity whereas causal pathways between holons establish system relations. These dual aspects of a system account for both the origin and behavior of a system. Whereas ultimate circular causality (wholeness) is implied by this form of analysis, it may or may not be evident in a given system or study.

Louie's point that the statement 'contains a closed path of efficient causation' does not mean completely 'closed to efficient causation' (Louie 2009, p231), is reflected in the holon framework. Natural systems can be simultaneously described as a circular hierarchy of causes establishing their identity, and a system of causal relations with other holons establishing their properties. A single, fully circular identity holon is an ideal entity that summarizes the effect of many relations extending throughout nature, and thus through many systems that might also be identified. It may label a distinct system or one that is systemic within others. The combination of implicit (ideal) circular causality (establishing identity) and causal closures occurring both within and between holons (conferring system properties) establishes the holarchy.

The direction of the large arrows (counter-clockwise) **in Fig. 4** represents the hierarchical effect of each cause, and it is not reversible.[3] Bottom-up and top-down causalities are two halves of the cycle, not opposite directions. Our ability to imagine the reverse hierarchy (to make inferences) would be described by a cognitive holon with its own causalities acting in the same direction. Later it will be shown how interacting hierarchies can account for different formalisms involving time and sequence. The difference between natural and cognitive systems in this theory is merely that they are different systems; the cognitive system is employed in an organismic system that is capable of utilizing its anticipatory nature for strategic and evolutionary advantages. Thus, an observer (or observer's experimental design) has its own formal causes that intersect with those already involved in the system, which may therefore result in selected complex effects.



Explanation of Quadrant Descriptions

- Process (*efficient cause*): decoding of functions into local reality, as a natural behavior.
- Event (*material cause*): encoding of natural system properties into structure, as a measured condition (in both space and time).
- Example (*final cause*): encoding of realized patterns from structure into a contextual system model, as a natural exemplar.
- Attractor (*formal cause*): decoding of contextual models into function, describable as a behavioral potential.

Figure 4: Holon defined as a natural modeling relation comprising the four

New definitions for Aristotle's causes are given below to allow them to be related as a holon in the above manner. Statements are provided to define the ontological and epistemological meanings. Those definitions are followed by a list of comparative characteristics.

1. **Efficient cause:** (1) process, action, or change reconfiguring a natural system and thereby realizing a function; (2) behavior of a realized system; (3) laws of change inferred from behavior.

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[3] In previous work by this author a clockwise arrow was drawn to indicate our ability to infer causality; but in the light of the analytical framework described here, it seems more appropriate to remove it.

    a. The result of efficient cause is a realized natural system defined and organized by processes. Efficient cause is a consequence of function expressing formal cause and it informs material cause with natural referents.
    b. Efficient closure indicates a functional object; an agent of change.
    c. Hierarchically, efficient cause is the second part of decoding in a modeling relation and the first part of entailment in a realized system.

2. **Material cause:** (1) events, establishing properties of a natural system that are abstracted by interaction (including observation); (2) the local attributes abstracted as knowledge (states or structures); (3) observed or measured existence.
    a. The result of material cause is structure comprising interactive or descriptive properties that define and locate an event of a realized system (an instance of local existence). Material cause is a consequence of abstraction of a natural system and it informs final cause with examples.
    b. Material closure indicates a natural object; a material system with unique behavioral attributes.
    c. Hierarchically, material cause is the first part of encoding in a modeling relation and the second part of entailment in a realized system.

3. **Final cause:** (1) exemplification; that is, a system's use of prior phenomena to establish a goal or direction that may attract development of a system along a path to a defined end; (2) directional tendency toward a pre-defined result contextualized as an exemplar; (3) end-directedness built into a system model (a programmed result or goal direction).
    a. The result of final cause is a model of possible existence defined and organized by examples establishing potentials for existence (exemplars). Final cause is a consequence of contextualized structure and it informs formal cause with models.
    b. Final closure indicates an exemplary structural object or percept.
    c. Hierarchically, final cause is the second part of encoding in a modeling relation and the first part of entailment in a contextual system.

4. **Formal cause:** (1) attraction; that is, a potential that realizes a contextual model in terms of functions; (2) an encoded pattern, template, or general metrics governing laws of a system; (3) a formalism that defines functions and provides the terms of reference for natural phenomena and general metrics for laws of nature.
    a. The result of formal cause is function defined and organized within potentials that act as attractors. Formal cause is a consequence of model expression and it informs efficient cause with constitutional prameters and metrics.
    b. Formal closure indicates a final object; a selective template or model with implicit ends.
    c. Hierarchically, formal cause is the first part of decoding in a modeling relation and the second part of entailment in a contextual system.

Causal objects occur within holons at any level of the causal hierarchy (any quadrant of a holon). **Table 1** summarizes the hierarchical recursive association of the causes, where causal closure establishes an object with properties at the next higher level.

Table 1: Hierarchical Recursive Association of the Causes

Cause	Depends on	Closure Implies	Results in	Establishes
Efficient	Formal cause	Function	Changes	Material cause
Material	Efficient cause	Natural System	Properties	Final cause
Final	Material cause	Structure	Models	Formal cause
Formal	Final cause	Model	Tendencies	Efficient cause

Louie referenced Thomas Aquinas' idea of a 5th cause, called "exemplary cause" that is associated with anticipation (Louie 2009, pg.129). He described it as "a potentiality that anticipates another actuality, the becoming and being of the formal cause of something else" (Louie 2009, pg. 240). This description has two parts: First the idea of an exemplar and second the tension between potentiality and actuality. Final cause is described here as the application of an image of nature, drawn from nature, and guiding development toward that image as a consequence of potentials established by it and its contextual embedding. It is not, therefore, merely the end result of an efficient process; that end induces context to cause a similar end, and for that reason it is final cause. Final cause may thus qualify as Aquinas' exemplary cause in the sense that nature is continually manifesting according to examples of itself. Also, in a deeper sense more related to the relation between the causes themselves, we can see that causal closure at any level provides an instance or example of the next higher cause. This sense of 'exemplary causes' (plural) is also true; that the entire holon is built on hierarchical exemplification. However, we should be cautious associating final or exemplary cause with anticipation, as will be discussed in more detail later. More than just following an exemplar, or exemplifying higher causes, anticipation follows exemplars that are meaningful over time, as anticipation in organisms has ultimately to do with sustaining their functions. The ability of a system to change into a different kind of system by changing its function in nature, and thus to change its suitability for existence, is a new kind of behavior that is more than a mechanism and more than merely complex; anticipation involves an adaptive model. This quality does not arise from any single cause or from a single holon, but from a closed hierarchical composition of holons that form an M-R system.

Also, there is no need to invoke 'immanent causation' (Louie 2009, pg. 127) as fundamentally different from the causal hierarchy of the holon (except to imply a further transcendent unity that we cannot presently touch with theory). We have already seen that no number (or only an infinite number) of efficient causes can get us to complexity. Instead, complexity is the natural consequence of formal cause differences that organize efficient processes in multiple ways (see Rosen 1991, pg. 237-238, and Fig. 9F.4). If we accept that nature models itself, final cause also remains within nature's entailments, and there is no theoretical need for it to be mystical. Final/formal causation, the role of natural context, can thus account for immanent causation when we consider that psychological exemplars can come from different contextual domains that are involved with cognitive functions in the organism. The necessary causes for that are 'outside' of nature only in the sense of being prior to externally measurable events, but they are entirely inside nature and material systems with regard to their contextual definitions. It is not the aim here to speculate on the mind-brain problem, but one theory holds that ‘space-time selections’ can occur in the microstructures of nerve cells, which form a sufficiently isolated quantum context (Hameroff and Penrose 1996). In other words, alternative space-time domains can exist in this way and their selectivity may underlie consciousness.

Any system of description, however, must be incomplete at some level, and therefore it must ultimately have an unexplained origin. In mechanistic theory that origin is necessarily outside the system, but in relational theory it is inside. The relational ‘big bang’ is any event that establishes a difference between foreground and background, as opposed to a cosmic explosion of everything from nothingness. Technically, as a system attempting to explain itself, mechanism implies a ‘liar’s paradox’, because by the very meaning of a defined system its definition must exist in something else, implying an infinite series of definitions and their incompleteness. An infinite sum of mechanisms, supposed to arrive at complexity, actually gets us no closer. The relational holon and its complementarity between potential and actual may be the closest we can come to comprehending the origin of systems.

The relation between potential and actual has been at the root of our deepest and most ancient philosophical thoughts. To ancient Vedic scholars it represented dual aspects of unknowable eternal essence (Brahman), forever engaged in a cycle of self creation, with two ‘full’ but opposite aspects of reality originating each other as “existence with attributes” and “existence without attributes” (Mehta, 1970). We can reason that potential must act, and action must potentiate in such a relation; and that may be as much as can be said about the underlying cause of the relational holon itself. It is, like Brahman, “without cause”, but with “four faces” (Muller 1884). Here, this basic duality, which is also a non-duality, is given a mathematical form and analytical application. Modern philosophers have also said that foregrounds and backgrounds must be in communication or else neither could be experienced. Hence formal and natural systems cannot be said to exist except by mutual implication and realization, and this principle can be made scientific by entailing that relation within nature. The profound implications of this solution, and the historical confusion over the recursive nature of the four causes, are undoubtedly among the reasons that this step was not completed earlier.

3. Holons in Category Theory

Mathematical representation of relational theory is essential to make it rigorous and to establish it as an analytical method. However, as currently developed, relational theory employs category theory only to represent half of the holon, using efficient closure to indicate complexity but not representing the additional causes involved. The objective now is to give the modeling relation a complete algebraic representation in category theory, as a holon with a four cause recursive hierarchy that is explicitly both whole and part. We can see the means for doing that most clearly by re-examining the idea of self-entailment and identity.

3.1. Self-Entailment Revisited

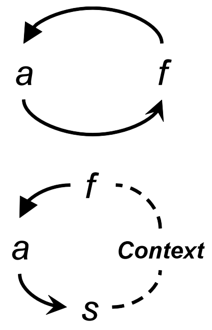

Figure 5:
Self-entailment

Using conventional notation (more appropriate symbols will be introduced later), a self-entailment mapping, **f** (**Fig. 5**, top), of **a** (representing a natural system) onto its own generating function, **f**, is paradoxical in the traditional graphical algebra of category theory (Louie 2009, pg. 116)[4] and is therefore considered to violate the logic of a simple efficient cause system. However, the real problem is that it is an incomplete picture of the relation; it does not show how the result of a material map (an open headed arrow from **a**) is transformed into **f**, the beginning of an efficient map (a solid headed arrow). Alternatively, if we show the efficient/material mapping as normally represented (left side of bottom diagram in **Fig. 5**) there is a result, **s,** in the co-domain (representing structure) that must be distinguished from **f**. Hence there is an implied complementary entailment (right side, with dashed lines indicating the contextual maps) needed to achieve closure. The implied entailment must be the natural inverse or natural complement[5] of the realized mapping, in which **s** becomes a morphism and **f** is its result; and it occurs in the contextual definition of a system (the right side of the holon), in contrast with realized mappings on the left side. Together, these two complementary categories form a modeling relation (a contextual relation), which is a holon involving all four causes, as shown in **Fig. 4**. In its simplest form, the holon thus provides the means for representing system identity.

Both Rosen and Louie argued that the condition for complexity is established by a "closed path of efficient causation" which cannot exist within a purely efficient (i.e., simple) system. Louie concluded that "causal entailment patterns with and without closed paths of efficient causation are different in kind and the barrier between the two classes is 'non-porous": [and quoting Rosen] "there are no purely syntactic operations that will take us across the barrier at all" (Louie 2009, pg. 232). This statement does not mean, however, that there are two realities, one that is simple and one that is complex. The duality is between two complementary domains of reality not between two kinds of natural system, and its implicit 'porosity' is not purely syntactic because it involves the higher causes, which are semantic. The statement that efficient closure is impossible in a simple system thus means that there are no completely simple systems in nature, just as there are no completely isolated identities. Simplicity is a constraint or reduction of the complex, whereas identity is an idealization of it. Our language emphasizes the reality of simple objects because it developed from perceptual experience, and it has consequently approached complexity as though it must be emergent; but it is the simple world that is emergent and in which complexity can only be seen as 'complication' (Rosen, 1991). The more parsimonious view is that complex systems are the fundamental reality and simple

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[4] The diagram is reversed from Louie's presentation to correspond with the holon diagram in Fig. 4.

[5] The terms 'inverse' and 'complement' have other mathematical definitions; hence the term 'natural inverse' or 'natural complement' are used here to refer to the case where natural structures are represented as functors on a contextual category that entails functions.

systems emerge from abstractions that result from interaction. Both interaction and perception thus simplify a natural system by abstracting properties from it.

We may equally say that we can know of no completely contextual systems; that is, system models without realizations, because they are known through their realizations. Still, their implicit nature does not mean that contextual causes can be ignored or reduced, because they are valid inferences from the nature and existence of consequent interactions. Self-entailment should also not be considered impossible nor trivial in its holistic form. The identity holon serves as a template for describing any type of system, whether simple, complex, or living, with qualities that arise from the nature of context (formal cause) and the way such identities are combined (compositional types). Just as the material particle was the idealized unit of classical mechanistic theory, the identity holon is the idealized unit of relational theory.

There is nothing mysterious about relational analysis if we accept that nature is established by self-reference. We have not been accustomed in modern science to asking why laws are consistent throughout space and time or how that consistency (or predictable variation) is maintained. Mechanism assumes it is so, but relational theory says it is only relatively so. Therefore, the analysis of causes must extend outside the local frame of reference of space-time efficiency and material abstraction to allow complexity to be studied. Consequently, even when a closed loop of entailment involving multiple efficient maps is proposed in relational theory, as in Rosen's M-R system diagram (Rosen 1991, Chapter 10) or Louie's corresponding entailment maps (Louie 2009, pg. 126), the 'head to tail' construction between efficient mappings must be taken as a shorthand, implicitly involving a formal cause context in each hierarchical composition. Even a single efficient map implies identity and therefore context.

Rosen's M-R entailment diagram must also be seen as a shorthand that shows the net effect of hidden relations expressed as closure of efficient causes. Without contextual analysis it may seem to be a paradoxical arrangement. Efficient composition diagrams that do not represent the contextual category at all offer no way to analyze the condition of complexity or ontological implications of context (i.e., issues of origin). We can see, for example, that a 'circular hierarchical entailment' of efficient causes (Louie 2009, pg. 222-226), which will be discussed in greater detail later, is an indicator of complexity, but not a description of complexity. In other words, there is an equivocation that must be resolved between a closed cycle of entailments that cannot exist when interpreted literally as efficient/material cause (Louie 2009, pg. 114, 116, Theorem 8.44 and 9.3 on pg. 222), and the definition of complexity in the same terms, (Louie 2009, pg. 149 and 229-230). Adding the description of contextual causes resolves this issue because it is precisely the ability for causal relations to exist separately from realization that allows the result of an efficient mapping to entail its own or another generating function.

3.2. The New Algebra

Restating the conclusions above, complexity is explained by complete four-cause holon relations, which establish a natural recursion between realized and contextualized existence of a system; and implicitly all four causes must be present in any composition of causes (sequential or hierarchical). We therefore need to define causal maps in such a way that they can distinguish between the four causes and relate them as a recursive hierarchy (which, in multiple compositions, is a holarchy).

If a morphism, **f** (in the common notation), has domain **A** and co-domain **B**, with elements **a** and **b** respectively, it is traditional to→write **f : a** → **b** , or **f: A** **B**, using an arrow to denote the map between elements **a** and **b** or between domains **A** and **B** (or to use the two-arrow form in **Fig. 5**), the former tracing elements of the domains and the latter relating the domains. While such symbolic maps can be applied generally to any mapping, we need a way to distinguish the very specific and complementary meanings of realized and contextual causes described above, retaining their relation in the causal hierarchy. That distinction, made below, is what then allows the further distinction and definition of structure and function as epistemological elements in an analytical framework.

In category theory, 'categories' consist of both 'objects', and 'morphisms'. A morphism and its mapping are denoted with arrows or equivalent algebraic symbols, where morphisms map objects (which may be 'structured sets' or categories themselves) between a source 'domain' and target 'co-domain'. As a general foundation for mathematics and computer science, category theory allows objects and morphisms to be defined very flexibly, according to application. Here the aim is to apply these concepts to relational theory, which is much more specific about how nature should be entailed in terms of natural structure and function as part of modeling relations. The holarchical property of natural systems is represented in the case of categories of categories, in which the roles of morphism and functor become analogous. Holons may then be treated as special kinds of objects, which are dualistic and infinitely holarchical.

Specifically, we must define two categories and their associated mappings corresponding to the left and right side of the holon; that is, categories representing the dualism between locally realized and non-locally contextualized causes. The two categories are thus required to be natural inverses (natural complements) of each other, not in the sense of different mappings that exchange domain and co-domain, but in the sense of different mappings that exchange the roles of co-domain and morphism. The definition of objects and morphisms will therefore be different in each of these categories. Louie perceptively described formal cause entailment as a coincidence of 'immanence' and 'imminence' with meanings of inherent vs. impending existence, without a doubt the ontological duality described here and the underlying principle of complexity; that is, relation between realization and contextualization.

The general form of categorical mapping (the category of realized transformations that we are familiar with in mechanistic science) corresponds with the realized morphism of efficient entailment in relational theory (as employed by both Rosen and Louie). That kind of morphism can be used to describe how a natural function entails a natural structure via the realized domain. However, we do not have corresponding language and algebra for the category of contextual transformations introduced here; morphisms that describe how a structure entails a function via the contextual domain. We should therefore define a contextual morphism, perhaps using the term structor as the natural inverse of a functor. This language would allow us to associate the term functor with natural functions, as dynamic agents decoded from context and accounting for measurable states and their differences (change); and similarly to associate the term structor with natural structures, as attractive exemplars encoded into context to form models.

Natural structure and function bridge the realized and contextual categories. They act both between them in an analogous way to structors and functors, and within them in an analogous way to contextualized and realized morphisms. For example, the natural function 'feeding' may represent a general ecosystem potential for feeding to occur, thus relating between general categories; or 'feeding' may specify an organism's feeding behavior and strategy, thus identifying specific processes. In these ways 'feeding' may be treated as a generic or specific object of analysis that may occur in various causal chains describing its origin or its effect. Similarly, in this example, the state of resources that results from feeding can also be treated as an autonomous object of analysis that acts through various causal chains to induce other functions. In a relational sense, the contextual potential for occurrence of a phenomenon is no less a reality than the phenomenon; they establish each other's reality.

Fig. 6 presents an expanded algebra for applying category theory to distinguish contextual entailment as a natural inverse complementary mapping. These new conventions provide sufficient detail to distinguish realized and contextual maps and to thus integrate relational mapping with modeling relations. It is a relatively straightforward application of category theory terms to specify the dualistic relation discussed above.

With this notation we can manipulate the natural concepts of structure and function as complementary mathematical objects representing epistemological abstractions of the natural and contextual domains of the holon, respectively. Also, the unity of these two basic categories gives us an analytical concept of wholeness, where the realized and contextual categories correspond with externally and internally defined properties of natural systems, respectively. Accordingly, realized morphisms can present functions entailing structures as sets of measurable properties (characterizing natural systems via external observation or interaction); and contextual morphisms can present structures entailing functions as sets of contextual attractors (characterizing system context as a distribution of potentials). Thus we can specify contextual mappings that serve to close efficient causes and impart specific properties of the context (such as temporal sequence, uncertainty, relativity, adaptive potentiality, or other properties of formal cause).

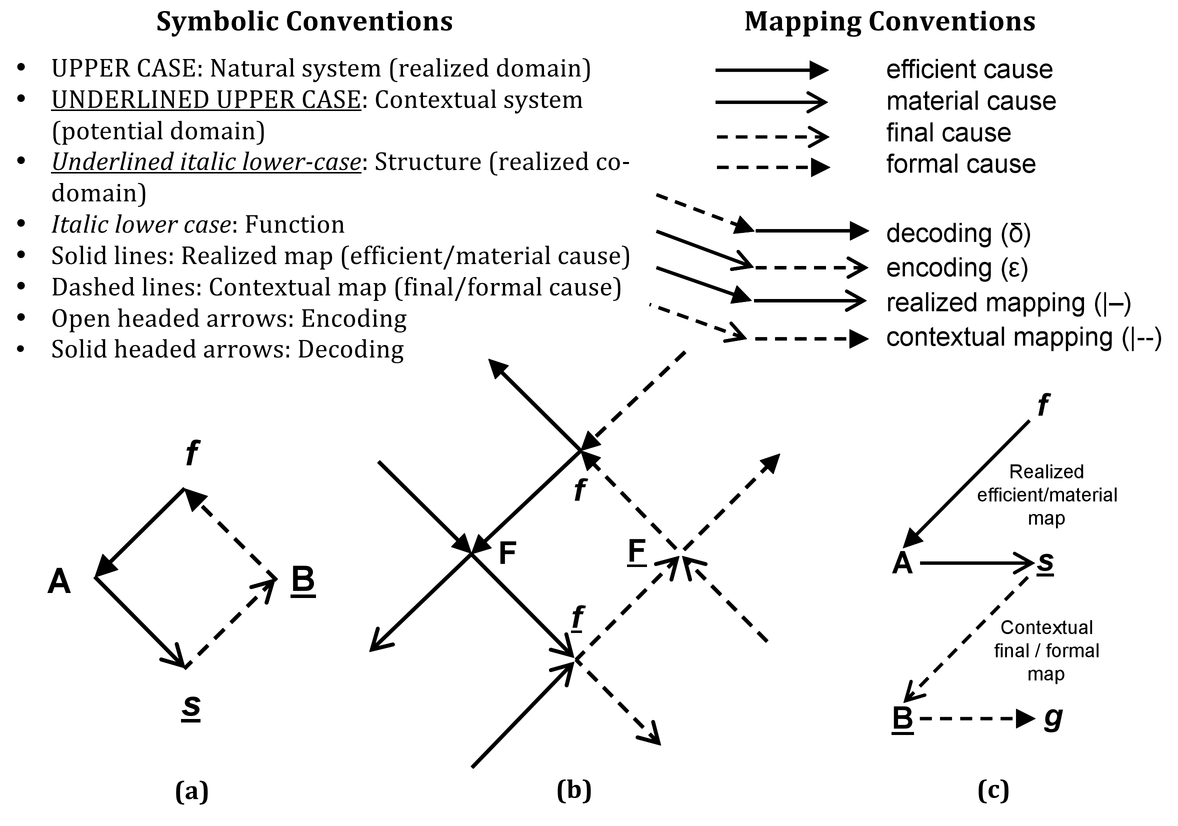


Figure 6: Proposed Relational Holon Algebra. **(a)** Self-entailment of ***f*** via extrinsic components. **(b)** Identity holon showing intrinsic and extrinsic relations. **(c)** Holistic composition of

New symbolic and mapping conventions are given (top of **Fig. 6**) for labeling causal arrows and nodes of a mapping (beginning and ending of an arrow) to distinguish between their realized and contextual counterparts and to equivalence the encoding and decoding arrows of a modeling relation with corresponding arrows in category theory. The causal hierarchy of the holon presented earlier in **Fig. 4** restricts combinations of causes to a recursive four-part hierarchy and therefore four possible pair-wise combinations (arrow conventions shown in **Fig. 6**). Two of these combinations correspond to encoding and decoding of modeling relations (epistemology), and the other two correspond with realized and contextual entailments presumed of nature (ontology). Entailment symbols, which are a useful shorthand for causal mappings, are similarly distinguished by adding a dashed-line symbol (|--) to summarize the contextual category mapping in the same way that the standard entailment symbol (|–) summarizes the realized category mapping. Accordingly, these symbols must alternate in series in any entailment diagram. The two causal pairs corresponding with encoding and decoding are distinguished by the symbols **ε** and **δ**, respectively.

These conventions and their primitive constructions allow holon types to be defined for analytical purposes (the three diagrams in the bottom half of the figure). Diagrams **6.1** and **6.2** show the simultaneous ways that self-entailment (as explored in **Fig. 5**) can be diagrammed in accord with the assumptions and conventions here. In diagram **6.1**, **f** entails itself through relations with systems that are not identified with the **F**-holon (the capitalized domain label identifies the holon) and therefore are (by definition) extrinsically involved in entailing **f** with itself. Each symbol in the diagram implies another system and thus another holon. This case appears more specifically in Rosen's M-R system diagram where four holons are related. Diagram **6.2** shows the identity holon for system **F** as discussed in Sect. 3.1. and as it might be shown for any identified system or component. We can now see that the identity holon itself is an idealization of nature that is known only by its external compositions. In keeping with its true whole/part nature, it simultaneously has pathways of external relation as indicated. Both kinds of relation can establish a circular hierarchy of relational causes indicating the existence of the **F**-system; in one case looking at **F** as a result of external relations, and in the other case looking at it as a uniquely identified system that is causally whole. Diagram **6.1** does not distinguish between internal and external physical components of a system; higher order closure does that by establishing system boundaries that may be realized. We thus have the option of representing **F** or any of its related systems, with respect to intrinsic or extrinsic causes, or both. The detailed compositions of holons are of analytical interest, whereas the identity holons label systems. The apparent equivocation between identity and interactive relations is essential to the holon's simultaneous whole/part nature and we must consider both views as simultaneously true.

Identity is thus a circular pairing of realized and contextual entailments. It is a single recursive causal hierarchy where each of the four components of the holon (system, context, structure, and function) form a closed loop defining a system (giving it identity). Since relational analysis is holarchical, these components and their causalities may also be expanded with regard to their external or internal causes. Holons may thus be embedded in or composed of other holons. For this reason, while an alphanumeric symbol can be used to suggest the causal role of an element of the mapping, such as '**s**' for structure and '**f**' for function (as in **diagrams 6.1** and **6.3,** and later in **Fig. 9** showing structure-function relations), more detail can be given by adopting system labels that keep track of identities. In either case it is convenient to denote the causal role of each symbol by use of formatting conventions (upper left part of the diagram). For example, system **A,** as shown**,** consists of four causal objects labeled as **a**, **A**, $\underline{\mathbf{A}}$, and $\underline{\mathbf{a}}$. When relating different holons, the symbols can thus be used to keep track of the system identities while their formatting signifies which aspect of the holon is being related. These conventions allow the choice of tracing system identities or causal relations between identities, as needed.

Diagram **6.3** shows how realized and contextual entailments compose generally between holons. For example, the closure shown in diagram **6.1** might extend through many systems. In that case diagram **6.3** represents a link in such a chain, an open holon, or holism. Its compositions imply wholeness, but it is not itself complete. The corresponding text notation would be written as ***f*****: A →** $\underline{\mathbf{s}}$ (the efficient mapping) and $\underline{\mathbf{s}}$**:** $\underline{\mathbf{B}}$ **→ g** (the contextual mapping), or more summarily, **f |–** $\underline{\mathbf{s}}$ **|-- g**. Thus ***f*** is a function that entails the structure, $\underline{\mathbf{s}}$, from the natural system, **A**; and $\underline{\mathbf{s}}$ is a structure that entails the function, **g**, through a context, $\underline{\mathbf{B}}$. The mappings are implicitly expandable by specifying the identity holons for each of these labels, which then leads to consideration of their extrinsic relations passing through different holons. Holon identity and interrelation are complementary aspects of a system and alternative ways of representing its characteristics at the level of detail required in a given problem. We can thus analyze the complexity of a system without losing its implicit wholeness by explicitly retaining the duality of identity and relation.

We saw that contextual mappings must be distinguished symbolically from realized mappings because they occur within an inverse category of entailments that have no equivalent representation in the realized domain. They represent co-occurring natural models that act together to determine a potential, but they do not interact as discrete systems or properties. The realized aspect of a system, on the other hand, is defined by interactions between exclusive (spatially and temporally distinct) properties and therefore is characterized by efficient and material dynamics in space and time. Whereas the properties of the realized domain are accumulative, the properties of the contextual domain are selective.

Incorporating contextual relations into the relational mapping algebra thus resolves the issue posed in the introduction, of how an inertial structure becomes a gravitational function in a 'head-to-tail' mapping composition; it does so by establishing a contextual model. Abstractions of a natural system acquire the holistic properties of the contextual system they enter, creating a new natural system that reflects both the structural properties originally abstracted, and the nature of the context. Structure is translated into function in contextual terms.

The distinction between elements and domains, as reflected in the capitalization and lower-case notation in the diagram, is also crucial and departs from the traditional interpretation that efficient cause is a mapping between defined elements. As long as the realized and contextual domains are interpreted as ontological aspects of natural systems, as they are here, they cannot be seen as containing pre-defined elements because relational theory must assume that the only knowable elements are those abstracted from otherwise un-described realities. The knowable elements of a given domain that correspond to a natural system are the abstractions: structure and function. In other words, the results of a mapping are epistemological, but the domain of a mapping must be considered ontological. We will see in Sect. 3.3 that compositions of maps that would take us between realized states are actually recursive holon sequences. Hence, the idea that states correspond to actual particulate elements pre-existing in a natural domain with determined properties must be abandoned. Instead, states are seen as jointly determined and ordered by the mutual causalities of system and context. 'Structure-preserving' metrics (in the mathematical sense of the term 'structure') are provided by the complementarity between realized and contextual domains.

From these arguments one may reason that the holon is a complex entity with respect to external contexts, and a simple entity with respect to its own context. The knowable world seems to exist between these idealized limits, retaining both principles. As a completely isolated and self-defined entity it could not be said to have properties, and as a completely interactive entity it could not be said to have identity or origin. What we perceive as existing are various generalizations or objectifications of the complex, and perception itself is an interrelation between holons that have both aspects. This characteristic of holons is, of course, precisely the 'participatory' aspect of nature that the early quantum physicists discovered, that naturalists intuited, and that we wish to describe more generally of complex and living systems. We could perhaps classify knowable systems by their degree of relative interaction and isolation, for example defining a continuum of system types such as mechanistic, uncertain, complex, subtle, and imaginary. Machines are at one extreme, characterized by efficient interactions, and perhaps we can say that experiences are at the other extreme, characterized by holon identity. Biological systems comprise the full continuum, and are arguably the most whole systems in the sense that they are equally characterized by both realized and contextualized aspects. For the same reason, they may be the most appropriate systems from which to learn the full range of causality.

Some deep principles of ecology that have not had a generally accepted formalism are implied in these relations. The necessity of shared holon interrelations ensures that 'everything is connected to everything else' because holons are partly shared entities. Furthermore, 'the whole is greater than the sum of its parts' for the same reason; each interrelation adds an extended causality to the holon. Although mechanistic aspects of interactive holons can be identified, identity holons are not necessarily reductions in the sense of being mechanistic; they summarize broader relations that may be complex or simplified, but that nevertheless identify a system as a natural object. As we have seen, the holon must have shared relations to be observable or to interact in a universe, and in that sense each holon has at least rudimentary non-mechanistic properties. In the same sense that a part is abstracted from a whole, a whole comprises an infinite number of potential parts. As a diagram of self-relation, the holon is also a theoretical representation of autopoiesis. A realized autopoietic system (Maturana and Varela 1980) has the same basic property of simultaneous wholeness and particulate interaction, which can be represented by the identity holon and its shared causalities.

The necessity of contextual relations in both identity holons and interrelated holons ensures that any composition of efficient/material maps alone will be an incomplete description of a system, ignoring its fundamental complexity. We thus arrive incontrovertibly at the need to add contextual maps to the analysis of system complexity. We can say that: Complexity is a natural condition that cannot be described by any composition of efficient/material maps without the intervention of contextual (final/formal) maps, and thus the necessity of their combination, which is the holon. Nature is thus describable in terms of systems that comprise whole, four-cause relations and entailments. We can now apply these definitions to complete the synthesis mathematically, and thus to establish a relational analysis method in terms of holons and their causal maps.

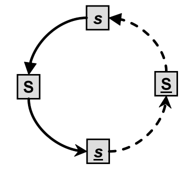
3.3. Holon Compositions and Closure

Given the fundamental nature of holons, the current diagrams for sequential, hierarchical, and circular compositions (Louie 2009, pgs. 117-127), which show only efficient and material aspects of a system, must each be modified according to the conventions in section 3.2 to add contextual relations. Any composition that was allowed in the former convention can be represented in this new protocol by showing its contextual maps. For consistency, we can consider the previous (efficient) notation to be another form of shorthand for holon organization, where the contextual relations are hidden. One may similarly show other portions of holons that comprise interrelations of interest; while all four causes remain implicitly involved.

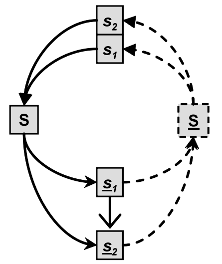
Multiple paths of causation between and within holons are characteristic of general complexity and nature. In fact, it is rare that an event can be explained by the action of a single cause, or that an activity in nature can be said to have only one effect. Furthermore, it may be necessary to model multiple paths of causation before knowing if complex components can be reduced to simpler models. Multiple cause and effect relations between holons may occur and be described at any causal level (in any quadrant of the holon), and the analysis must therefore allow joining and branching between and within holons, as shown in **Fig. 6**. Implicitly, causal wholeness may be realized in sub-systems or diffused through many shared relations. Despite the dual whole/part nature of the holon, as both an isolated and shared entity, the mapping remains logically consistent because, in principle, complete holons and their relations can always be traced and referenced to nature. Wholeness is therefore embedded in nature and it becomes apparent in systems that are sufficiently isolated from the more general causality.

Fig. 7 shows three basic constructible holon diagrams: identity (top), sequential (middle), and hierarchical (bottom). Here, curved arrows are used in an equivalent way to the conventions in **Fig. 6,** which will make it easier later to show contextual relations. Each diagram is shown in a circular form for discussion, but implicitly and dualistically each is also open to external interrelations as discussed above. The identity holon (top diagram) is the ideal circular loop of causal entailment, as introduced in Sect. **3.1**. Identity is a key concept in relational thinking because it defines what is meant by a system with characteristic properties, functions, and formative context. It defines the most basic kind of causal closure, which is an ideal closure of all four causes (as in **Fig. 4**) that might be called first-order closure. Given that holons may describe component and systemic relations, the identity holon identifies implicate wholeness anywhere in nature. Identities are thus emergent properties of relations that form a circular hierarchy and which can be labeled as a system and analyzed in terms of its behavior and origin.

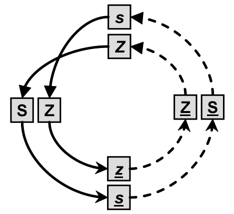
Simplicity corresponds with first-order closure at a universal level (i.e., applying to both system and environment), which is therefore a general reduction to a single implied formal system (natural law). Simplicity defines realization of mechanistic properties that conform with an ideal set of universal laws imagined to be syntactically complete (as in the classical world view). Further relations within that view, which are similarly reduced, may be described with or without contextual intervention because, by definition, the context does not vary in that common formalism. It is thus appropriate to describe simple causality using efficient/material entailments alone, if that results in a sufficient approximation. All systems must have such reduced properties in this ideal material sense, or else they could not participate interactively or observationally; it is the basis of the perceptual world. However, naturally complex systems, which all systems are at some level of detail, are not restricted to a single context or thus describable by a single complete formalism. Analytically, their description must involve dual identities, because a complex system is both interactive and autonomous at the same time. The component of that duality that is not directly involved in external realization represents the internal causality of the system, which is the basis for its impredicativity. The system is complex to the extent that its internal causality is isolated from the realization context. Consequently, the ability for systems to interact defines a common frame of reference identified as environment, while their ability to have isolated causal identities defines an internal frame of reference with complex (impredicative) properties with respect to the environment. This duality between insides and collectively determined outsides will be extremely important in the analysis presented later.



Identity Holon for System 'S'



Sequential Composition



Recursive Hierarchical Composition (two systems, S and Z)

Figure 7: Basic holon Compositions

In Rosen's treatment of causal closure he was primarily discussing efficient closure of M-R systems as the minimal causal requirement for life forms. We will see in later sections that M-R systems represent a unique case of what may be called a fifth-order closure of the four causes; that is, closure involving five identity holons (if the environment is considered as one) and their functions, which internalize all four causalities. Second-order closure, which is the minimum causal requirement for a complex system (that it involve two distinct holons as a duality, and thus two contexts), is hierarchical composition (bottom diagram of **Fig. 7**). It ensures impredicative realizations (realizations that require more than one formalism to describe them) because it relates dual contexts that have irreducible formal causes as long as insides are distinguished from outsides. In the diagram, for example, we might imagine the **Z** holon summarizing internal causality of a system and the **S** holon summarizing external causality. Causal closure means that internal and external holons are mutually entailed and that information (modeling) relations exist between them.

The sequential composition (middle diagram **of Fig. 7**) represents an intermediate form of closure between a simple identity and a complex duality. It is the case of a simple system defined by a self-consistent mechanistic context (corresponding with Rosen's reference to a "largest computable system"). It therefore deals with the concept of predicative system change (e.g., temporal sequences). Our traditional idea is that a changed system retains its identity but for its configuration in space and time, which changes according to a predictable pattern. But if we consider the role of context in the same way as above, we must alter the way sequential compositions have been previously interpreted based on their exclusive presentation as efficient mappings (Louie, 2009, pg. 117-121). Specifically, a sequential efficient map (solid-headed solid arrow) cannot point to the result of a material map (hollow-headed solid arrow) because efficient cause acts on realized systems (the domain and beginning of a material map), not directly on its abstractions (the result of a material map in the associated co-domain). Entailing the result of an efficient entailment therefore must first involve a contextual relation through final and formal causes before a subsequent abstraction can be made. Consequently, there are four arrows involved in sequential entailment: a contextual entailment of function (final/formal entailment) and then a realization of that function and consequent entailment of a new, sequential structure (efficient/material entailment).

For an apparently simple system, it is a valid approximation to think of state transitions in terms of continuous change as specified by the system's sequential entailments. In that case a predicative context (the cumulative result of historical interactions, as discussed earlier) constrains natural system complexities to some degree of simple temporal dynamics, which is mechanistic formal cause. The presence of contextual relations is thus what governs the properties of a sequence; that is, the temporal order of system **S** shown as $\underline{\mathbf{s}}_1$ and $\underline{\mathbf{s}}_2$ in the diagram. These abstracted structures represent a difference between sequentially realized holon interactions. They can be summarized as continuous ('structure-preserving' in the mathematical sense) transitions because their contextual maps are homomorphic, thus the function in each recursion has the same form, which preserves the temporal order. If that were not so, the diagram would need to show contextual difference, in which case it would form a hierarchical composition. An apparently computable change should therefore not be imagined as taking place between fully determined natural structures, nor as being randomly produced, but as being abstracted sequentially according to the pattern specified by contextual models.

This interpretation of sequential cause, where the sequential composition is a reduction of complexity to temporality, implies that reality has a discrete (non-continuous) nature, organized by a formal system that may provide varying or even discontinuous metrics (as, for example, at scales where the relativistic difference between space-time frames of reference is apparent). The frequency and temporal phase between such frames of reference for observation and other defining interactions may then matter in observed behavior, as the communication between foreground events and background potentials varies each context. Obviously a great deal can happen between measured events if the metrics governing relative occurrence are recursively defined by the events realized. We thus obtain a discrete view of change in which existence between events does not have states at all, only potentials for existence organized recursively by formal domains that may differ in their organization.

The closed hierarchical composition (bottom) defines a relationship between two or more systems and accordingly the same number of contextual systems, shown here as <u>**S**</u> and <u>**Z**</u>. Implicitly, these contexts are not equivalent, making the system a complex, second-order closure. However, to the extent that these contexts have a combined realization (to the extent that they 'overlap', so to speak, as will be seen in Sect. 3.4**.**), the hybrid system may have simpler realizations (with sequential entailments as described above) that allow it to interact with other systems. Such relations constitute the mechanistic aspects of nature; they are simpler fractions of a complex system. So, according to these arguments, a natural system has both characteristics; if it has a realization, it must be describable as both hierarchy and sequence. Therefore, the existence of 'closed cycles of efficient causation' defines complexity if contextual differences between cycles are impredicative, and it defines temporality otherwise. This criterion for complexity is more stringent than causal closure alone, which includes the first-order closure of identities and the intermediate case of sequential entailment. The circular hierarchical composition implicates complexity because it specifies distinct contextual domains that differ by more than a space-time sequence, and thus allows complex ways for the separately realized effects of those domains to interact. We cannot remove the ambiguity, which is an analytical choice, in which two related systems, such as **S** and **Z**, can also be represented by a single larger holon with simple or complex context. Nevertheless, interaction establishes simplicity whereas closure establishes complexity.

Two hierarchically related systems, **S** and **Z** (as in the diagram) can have different temporal metrics in their respective sequential compositions (their space-time realizations) to the extent that they remain causally isolated from each other and larger contexts that would otherwise bring them into correspondence. In that case, **S** and **Z** can model each other in different time frames through both hierarchical and sequential composition. Arguably, if we combine hierarchical and sequential maps this way, it may be possible to describe complex sequences in time, including retro-causation, where two systems have causal relations on different time scales. Rosen's concept of an "internal predictive model" by which organisms may anticipate the future (and, presumably, consider the past) is also explained in this way; that the internal model operates on a different time scale, running forward at a faster rate to anticipate the future, or running slower or even backward to consider the past. Any physical system would have at least subtle complexity of this kind at scales where the discontinuity between abstractions is noticeable, as in quantum and relativistic systems. In relativistic systems, both localized and distributed properties of space-time (the way space-time metrics change over time and distance) are also determined by formal cause. The continuous Lorentz/Minkowsky variation in the 'shape' of space-time can be described relationally as a self-similar construction of coordinates from realized events that form and follow a universal context (Kineman 2009a). Uncertain observations at the quantum level may thus be explained in relational terms as occurring between or prior to otherwise formative system interactions.

The concept of causal closure thus includes any kind of two-way causal pathway between holons, which generally establishes a surrogacy of one holon for all or part of another and implies many kinds of diagrams. Given, in this theory, that all causality is at some level completed as a holon, any causality must ultimately result in a form of closure; however, the closed pathway may not be known or of interest in a given analysis. Hence causalities that are not shown as closures can be drawn as causal fractions. In this sense the standard realized entailment map is a fraction of a holon.

The most basic kind of closure between systems (holons) is the exchange of a single cause at one hierarchical level (one quadrant of a holon) as in efficient closure. Such closure implicitly expands a causal arrow in a holon diagram into a complete holon. The second-order hierarchical composition in **Fig. 7** (bottom diagram) was shown as an efficient closure; that is, taking place across the efficient cause quadrants of two holons as an exchange of processes. As a result of such closure, each holon explains the efficient cause of the other. Closure between holons may theoretically occur in any quadrant, with the meaning that the complete **S** and **Z** systems are related by the characteristics of that causal property. Thus systems that are materially closed share matter and energy, if finally closed they share structural exemplars, and if formally closed they share attractors.

We saw that identity and inter-holon causalities must exist simultaneously as dual aspects of the holon (its definition as both part and whole). Closure can be neither complete nor absent and is therefore present to some degree given a deep enough analysis, or absent to some degree given the conditions that establish mechanistic relations. Furthermore, the holon is fully embedded in nature; the relations to the right and left are dual aspects of natural systems, not different kinds of whole systems. The details of an analysis will therefore depend as much on how systems are defined and the questions being asked as on the empirical nature of the relations.

3.4. Closure in multiple causes

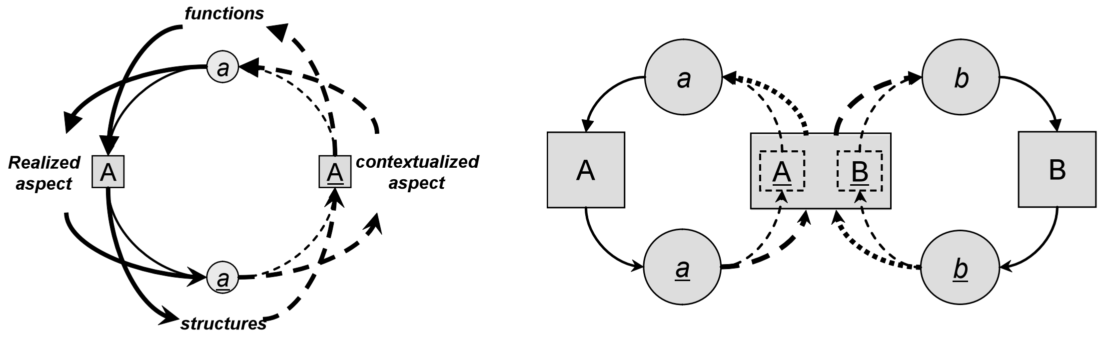
The more general case of dualistic holon causality underlying all relations and entailments (from **Fig. 6)** is re-drawn and re-labeled in **Fig. 8** (left diagram), replacing straight arrows with equivalent curved arrows in the style of modeling relations, as employed later in Sect. 4.

It is hard to imagine an organism or other functional system that has only single-quadrant closure, even though we may wish to focus on that for study. Indeed it may be that multiple quadrant closures are most meaningful. Two consecutive causes in the hierarchy form either an entailment (realized or contextualized category) or an information relation (encoding or decoding), either of which can provide closure between holons. A closed entailment can describe multiple functions or structures of a system, including, as we saw earlier, sequential entailments. A closed relation (which is a modeling relation) forms a chain, where holons share either a common system with multiple models, or a common model among multiple systems.

In all cases complexity is a matter of how the models combine (the order-preserving property of the combined models), so that the nature of system combinations ultimately depends on the association of contexts, as shown in **Fig. 8** (right side). In other words, even the circular hierarchical entailment can be described with more detail in this way, to show how its different holon labels are produced from contextual combinations. This is the case that also justifies statistical models, which can be diagrammed as a contextual combination in which complex relations are summarized as probabilities. This form of holon diagram, showing explicitly how contexts combine to form new, hybrid systems, is the key to describing complex relations, because complexity is explained nowhere else in the causal hierarchy but in the nature of context.

Contextual hybridization (which is final/formal closure) shows a very different phenomenon than combinations of realized systems or their structure-function relations. When systems share a common contextual domain, their realized properties co-distribute dynamically within the potentials of that domain. Their contexts intersect (as in Venn diagrams), and it is therefore possible for them to intersect irreversibly to form a new system. Due to the inverse association between contextual and realized categories, a mutual restriction (selection) between contextual models means a new possibility in the realized domain. The contextual domain is where one plus one may equal one, two, or three depending on the intersection. The case of identity (1+1=1) labels a common system that may be shared by the two original systems. The non-intersecting portions of the original models identify causally separate systems (1+1=2). Complexity occurs when the intersection implies a new system, as shown in the right-hand diagram in **Fig. 8**. This case is a generative closure (1+1=3), dealing with the origin of systems. Since the contextual combinations may have regions of intersection and non-intersection (continuing the analogy with Venn diagrams), systems **A** and **B** have all compositional types discussed in Sect. 3.3.

Of particular interest for the analysis that follows is the generative closure; the intersection of contextual models to form new ones, and thus the generation of new functions. That basic organization is the most parsimonious way to explain the M-R functions and their complex causal closure without borrowing additional contexts arbitrarily (and thus involving more systems). Any number of holons can be causally related by implicit closures in one or more causes, but in each case there must be hybrid contexts from which new functions (and implicitly new systems) are produced. The two systems, **A** and **B** in the diagram, each have their own implicit identity indicated by their identity holons, but additionally they are shown exchanging structures and generating each other's functions through mutual contextual entailments. A hybridized contextual model is established from the intersection of **A** and **B**. As a result we can show how **a** entails **b** and **b** entails **a** uniquely (aside from their identities), thus closing the two systems in final and formal cause and thereby limiting the possible system organizations to available contextual combinations. If the context of generated entailments is different from those of the identity holons, it is most parsimoniously the intersected **AB** context. By this logic, we may infer that M-R systems, which are explicitly closed in both efficient and material causes, also become closed (autonomous) entities in final and formal causes. In other words, we can show that M-R systems are whole systems (closed in all four causes) with respect to the internalized functions that Rosen identified and their necessary environmental entailment as discussed above. Furthermore, we will see that by analyzing such contextual combinations in a complex system, we can deduce the combinatorial possibilities for various system types.



System A identity holon with external causalities.

Two systems (A, B) mutually realized by a shared (AB) model.

Figure 8: Inter-holon Closure

There are many interpretations of these closures that are possible. For example, sequential closure may characterize biological development; generative closure might characterize adaptation, evolution, or generation of new functions; and hierarchical composition may characterize functional replacement of organisms in an ecosystem, or of components in an M-R diagram. The meaning of multiple quadrant closures in multiple holon compositions is obviously quite important and deserves further investigation.

Although there is much more that can be said, it should be clear at this point that the desired synthesis of the theory of modeling relations and category theory, if not demonstrated, is achievable in the manner presented above, and that such a synthesis provides a robust and general theory of nature. The most basic result of this synthesis is the unification of all four causes as holon relations, and their implicit natural inverse complementary mappings. Those relations describe the underlying organization of nature, from which living, non-living, and mixed kinds of systems (such as ecosystems) arise.

3.5. Realization

The treatment of contextual relations as part of holons addresses the 'problem of realization', that is, realization of models into the phenomenal world, which Rosen commented on extensively throughout his work (see especially Rosen 1999). As discussed earlier, potentials for occurrence of an event are distributed across the instances of variables defining that potential, and realization is thus a translation (decoding) from models of potential into natural behavior. However, it has two parts. A naturally complex system, **A**, is specified by two holons; one that realizes its instances as distinct space-time events with respect to other events, and one that independently realizes the conditions for its existence implied in the contextual model (the system's implicit suitability). Accordingly, there is a difference between the potential occurrence of an event and its realized occurrence. The potential specifies suitability in terms other than space and time, and thus serves as an attractor of space-time occurrence. The actual occurrences of the system in space-time are subject to temporal dynamics, and consequently may occupy locations irrespective of fundamental suitability, and may also recursively reconfigure that suitability.

Given the nature of sequential composition (Sect. 3.3), occurrences are defined in one context at the frequency of its realized interactions and they have complex behavior to the extent that interactions occur at a different frequency in another context. Consequently, complexity can be attributed to such phase differences, where the recursive cycle of realization and contextualization in different holons implies different realizations. Instances of **A** may then occur differently from potentials for **A**, thus giving rise to both indeterminacy and dynamics. Such dual realizations become predicative (generally-defined) as a consequence of their history of interactions, as the metrics of each contextual model (of the interacting systems) becomes co-defined by a common set of events. In other words, system measurements and existence potentials should equilibrate to the extent that internal and external contexts become mutually defined. This condition occurs, for example, in biology as a result of high selection pressure, or in particle physics as a result of frequent observation. In both cases, dynamic and contextual attractors establish each other as higher selection rates move the system toward equilibrium between potential and actual existence.

In a purely physical sense, it should also be clear that the involvement of contextual relations means that the local space-time coordinate system itself, which is defined in reference to events, must be realized by similar complex recursions; for space-time has no other exemplar than its own history. For example, in the quantum world, if **A** represents a causally isolated physical system (recalling that causal isolation can be the result of scale), the location of an event may be abstracted differently in the space-time metrics of a present observer or interaction, than its potential for realization as defined by prior events. The result is that less frequent measurements will reveal the uncertainty between these two frames of reference and isolated interactions can produce unexpected (anomalous) phenomena.

The principle of complex realization may be easiest to visualize with an ecological example. The location of an organism or distribution of a species as material events and causal agents can be measured in one set of demographic variables, but its potential for occurrence involves another set of variables in which the suitable conditions for its existence (the ecological niche) are defined. As a result, the location of an organism is partly out of equilibrium with respect to its suitability (fundamental niche), and that disequilibrium induces change as potential and actual distributions, in effect, define each other. The situation may also be interesting regarding entropy (which has been associated with order or, negatively, with information in both physical and biological distributions). Disequilibrium between foreground distribution and background potential is a lower entropy (more ordered) condition associated with more usable energy, as contextually defined.

Physical (non-living) system complexity is not different in this aspect of realization under complex conditions. It is well known, for example, that relativistic frames of reference differ precisely according to their space-time separation. That difference can also be modeled as self-reference (Kineman, 2010). The result is a specification of nature in terms of space, time, and information, which may be re-defined as a modeling relation that encodes and decodes systems in very much the way that Bateson defined information, as “a difference that makes a difference” (Bateson 1979); but more explicitly between realized and contextualized domains. As argued in Sect. 3.3, change should therefore be associated not with the idea of continuous alteration of a space-time object, but with fluctuation (as a result of contextual encoding and decoding) of the existence of an object and the suitability of the space in which it is realized.

To summarize: If measurable occurrence of a system’s events and the conditions for their occurrence are mutually defined over the course of recursive realizations, then the causal organization of a system in all four causes conditions its developmental or evolutionary characteristics, determining what kind of system it will be. The term realization means attribution of events (instances of a natural system) with physical form and location (occurrence), and attribution of those same events with contextually determined conditions for existence (the natural model of a system or event) in independently realized variables. Both attributions are required to establish any natural occurrence, whether simple, complex, or living, and accordingly this principle can be found at some level in all natural systems. The further distinction of a living system from a non-living system is not at this level, but rather at the level where internal entailment of the system components controls realization adaptively.

As will be seen, an M-R system is necessarily adaptive, in which case its models are anticipatory. As such, the development of realizations, how the system defines itself over time through behavioral adaptation and evolution, will have an anticipatory pattern;

whereas the development of a non-living system will not. While both types of system co-define their contextual domain (via holon relations) a living system will be capable of exploring functional traits that can be optimized and sustained through internal changes and expressed behavior. A non-living system, in contrast, will co-select stable interactions commensurate with its prior internal definition, as it does not have the means for inducing its own change. Again we see that complexity involves the coincidence of two system realizations, one defining an event and another defining its place in a context.

Having argued that models should be considered ubiquitous in nature as a consequence of how causes are related, we can now consider the distinction between models and anticipatory models (Rosen 1985).

4. Interpreting Life Itself

Louie characterized the "spirit of relational biology" as the study of anticipation in terms of an "embedded internal predictive model itself" (Louie 2009, pg. 240). And yet, part of the message here has been that the living world shares the basic properties of complexity with the rest of nature; with fundamental implications for all of science. Modeling relations are obviously the bridge between living and non-living systems, and biology can be distinguish in terms of its additional entailments that build on the capabilities of modeling relations that are already embedded in nature.

What really distinguishes a biological system from a (merely) complex system is not a new kind of causality, nor even the presence of internal models, but the development and specialized use of models in adapting to predicted futures. Whereas it was argued above that all systems must be drawn toward realizations that correspond with the contexts they have in common with other systems, and complexity results from multiple recursive relations with such contexts; anticipatory systems have the further ability to select and be selected by those contexts, and thus to be drawn toward adaptations, increasing rather than decreasing the set of possible behaviors and altering both system and context (and consequently increasing order and information).

The important question for biology is thus how anticipatory models are created and used by a cell or organism. Biology is explained by organization in a special kind of closure that causes a system not only to be attracted to various exemplars, but to change its functions in an adaptive way that helps sustain the system and its functions. The essence of anticipation is captured in Rosen's idea of an "internal predictive model" if we understand that prediction must be combined with adaptation (behavioral or otherwise) for anticipation to be meaningful. Involvement of final cause in the recursive determination of nature as a consequence of ubiquitous modeling relations does not necessarily contribute to survival and evolution of a system without the capacity for the system to adapt. As we will see in the next section, the specific way that contextual models (and thus functions) are combined in Rosen's M-R systems makes them adaptive and thus anticipatory in unique ways as a consequence of their organization.

In the three sections that follow the new terms of this synthesis will be applied to reveal insights into Rosen's M-R systems. First, a limited structure-function epistemology that emerges from the holon relation will be applied to show that M-R systems are adaptive and therefore evolutionary systems. Second, it will be shown that M-R systems must have two distinct hybrid contexts to relate the three M-R functions, corresponding, as discussed above, to internal and external closures. Third, it will be shown that the contextual organization of an M-R system not only guarantees complexity, but implies three kinds of internal organization that may be related to the three basic types of cells (and thus organisms) found in nature. These examples are presented speculatively, as a demonstration of the tools of relational analysis introduced above, leaving further comment on their implications for elsewhere.

4.1. Structure-Function Diagrams

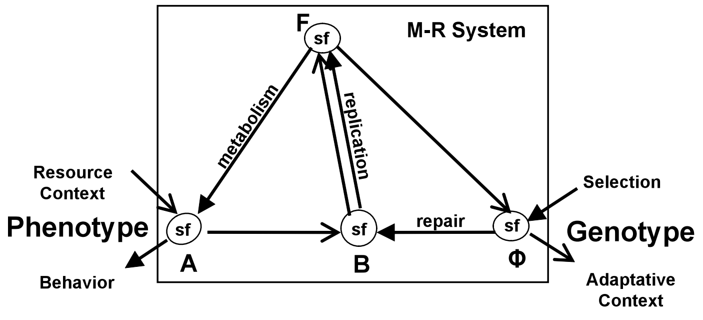


Figure 9: Structure-Function Relations in Rosen's M-R entailment diagram for living

Fig. 9 shows Rosen's M-R system entailment diagram (Rosen 1991), modified to represent each node as an implicit holon and component of the M-R system, with corresponding holon labels, **A**, **F**, **B**, and **Φ.** These holon components are responsible for producing the three internally closed functions that, according to Rosen, define a living system: metabolism, replication, and repair. Here we look only at the epistemological aspects of these components; how each interacts as a structural and functional (**sf**) unit. Each component must have both aspects and, disregarding the ontological aspects of the holon for the moment, we can summarily say that the nodes of the diagram are related to each other by functions that cause new structures and by structures that cause new functions.

As shown in **Fig. 9,** there are four pathways by which holons (nodes in the diagram) can be uniquely related by structure and function maps. Accordingly, each **sf** node relates to the others by means of pairing the function of one with the structure of another. The arrows in the diagram conform to the conventions for drawing efficient maps, and none of the contextual maps are shown. In this way, we can naively consider solid-headed arrows to be a functional cause of change in another holon's structure, and open-headed arrows to be a structural cause of change in another holon's function (contextual implications are examined in the next section).

Efficient closure of the three M-R functions is now apparent at the level of the entire diagram. The two nodes in the middle of the diagram, **F** and **B**, have all four causal pathways involved within the diagram's efficient/material closure, but at the other two nodes only two of the possible interactive pathways are part of the internal closure, leaving the other pathways to interact externally. Therefore, there is a structure associated with **Φ** that is produced but not modified within the diagram, and a remaining function associated with **A** that does not act within the diagram. These are free to act or be acted upon in modeling relations with the environment. They would necessarily be subject to adaptive and selective processes establishing them as phenotype and genotype. The phenotypic relation decodes behavior into the environment which in turn encodes material structures and information associated with resources (component **A** in the diagram). The genotypic relation begins with a structural encoding, presumably of genetic information represented by **Φ**, into an environmental genetic context (gene pool), which in turn decodes a selective function representing the contextual suitability of the organism. Thus, the more adaptive resource behaviors may be selected and encoded and internal models may be selected to represent various needs (e.g., food, nutrients, mates, etc.). If we consider the environment as one system in relation to the organism, the M-R system is a fifth-order closure (i.e., it involves a minimum of 5 holons in a circular hierarchical composition). When we consider the role of internal contexts, we will also see that the M-R system has closures in all four causes, thus identifying it as a whole causal object and agent.

The prediction of these known relations between organism and environment builds confidence in Rosen's original M-R diagram, but it also takes us one step farther. The next step is indicated in the relation between the diagram and its environment. Unavoidably, that relation must describe a potential for existence based on the realization of conditions that satisfy the organisms needs, while the organism's actual location and behavior results from its realized functions. The environment clearly acts as context in that complexity, at least with regard to fitness and selection, whereas the organism is contextual with respect to environmental modification. Only the realized maps are shown in this diagram; a phenotypic function decoding behavior and an environmental function decoding selection. However there are implicit contextual encodings: a structural encoding of resource properties into internal models of the organism, and a structural encoding of the organism's genetic properties into the environmental context, as the organism's fundamental niche. It is then clear that other organisms interact with the environment in the same way, and therefore interactions between organisms are mediated by the environmental context and can be described in terms of their respective niche potentials.[10] It is also clear that the organism (or cell) represented by the M-R diagram may itself be considered a single, complex holon with identity and interactive relations.

4.2. Contextual Block Diagrams

In the same way that context became a necessary consideration in analyzing the M-R entailments with environment, it is also important internally because organisms are compositions of such relations that have been internalized. Internal complexities may thus be analyzed by identifying the necessary contextual entailments of the internal M-R components.

An important aspect of contextual relation is that complex holons can achieve the same realized entailments with alternative contextual organizations. The degree of quadrant and holon closure for a given system (number of quadrants closed and number of holons specified) implies a discrete number of organizational possibilities. Consequently, there are different M-R system types corresponding to contextual organization alternatives. Significantly, contextual organization is not directly subject to the constraints of energy and entropy, which are properties of realization. By controlling aspects of the organism-environment relationship from the generation of new adaptive and anticipatory possibilities (for both system and environment) an increase in the number of contextual organization types in effect decreases entropy; that is it creates new order. Once again, this property may not be entirely unique to biology; it is a consequence of the inverse nature of the contextual domain and any degree of hierarchical closure, which necessarily implies a new contextual model identifying that relation. Nevertheless, as a consequence of M-R entailment, living systems seem to add a new level of control over these relations that are otherwise embedded in relational causality.

A closed relation between the three efficient maps and a common context, which we can now diagram as three modeling relations combined, is shown in **Fig. 10.** The realized portions of these systems are the three M-R functional entailments, metabolism (**m**), repair (**r**), and replication (**p**). Decoding and encoding are indicated at the top and bottom of the 'Contextual System' box and the coding arrows (structors and functors) are labeled according to (and redundant with) the mappings in each realized system. (Note that the path of relation between blocks in the diagram follows a three-way 'figure-eight' pattern). Each of the three realized system entailments relate in the same way to the implicit context of the system (normally the right side of a modeling relation). The implicit contextual entailments (that invert structors into functors) are represented by the upward pointing dashed arrow, thus indicating the first-order holon closures through which the fifth-order M-R system/environment closure is achieved. A more complete holon analysis is applied below to examine the contextual requirements separately; however at this stage we still learn more than was possible with just the structure-function diagram.

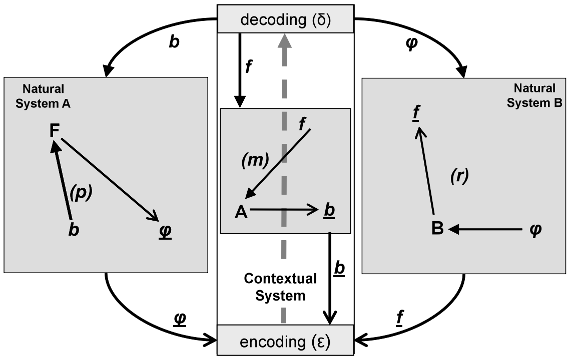


Figure 10: Contextual relation between entailments of an M-R

We know that both internal and external relations of the organism must be complex, and by the earlier arguments they should each be represented by a dualistic (hierarchical) composition. The block diagram shows how this criterion can be met, where two M-R functions may be internally contextualized while the remaining one (shown in the middle) may then be contextualized with the environment, thus having the role of establishing the organism's external relations. In this logical pairing of contexts, their combinations determine the alternative ways that the system can be organized, and as a result the constraints and opportunities available for behavior and evolution. This reasoning suggests an organizational explanation for the three kinds of life that are generally observed, where **Eukaryota** seem to be metabolically oriented systems with primitive to advanced resource strategies, **Bacteria** seem be characterized by a replication strategy, and **Archaea** are very robust entities possibly representing a repair strategy. The next task is therefore to look at the internal contextual relations to see if they describe this apparent coincidence in a manner that is consistent with the logic of circular hierarchical causes, using only the available elements in Rosen's diagram.

4.3. Holon Diagrams

The implications of involving two levels of contextual dualism in the M-R system, the necessity for which was described above, may now be considered in somewhat greater detail. **Fig. 11** shows complete holon diagrams for one of the three M-R organizational possibilities, now including the contextual organization discussed above. In this new kind of diagram, the interactive M-R functions are labeled explicitly as metabolism (**m**), repair (**r**), and replication (**p**) to distinguish them from the M-R component identity holons, **A**, **F**, **Φ**, and **B**, that they emerge from.[6] Otherwise, Rosen's original entailments are retained.

The left diagram shows how the right diagram was produced; by first drawing the identity relations, then adding the M-R entailments, and finally re-arranging the diagram to show necessary contextual combinations and implied components of the system. The cell/organism (M-R system) is thus described as a hierarchical composition of two systems that are themselves hierarchical compositions of the M-R components, in accordance with the relations shown in **Fig. 9**. One composition establishes internal complexity of the organism and the other establishes its external complexity. From the discussion surrounding **Fig. 6 and 7**, the aim here is to determine the most parsimonious organization of contextual domains of the original component identities that explain such

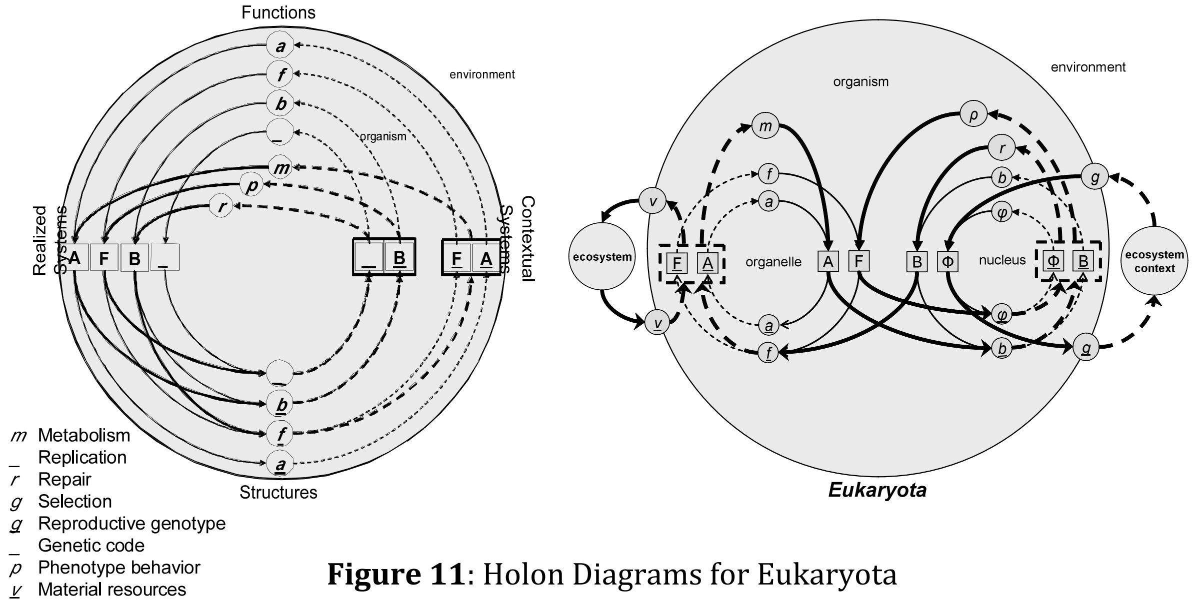


Figure 11: Holon Diagrams for Eukaryota

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[6] The implicit identity holons for the m, r, and p functions are redundant with the closed M-R holon elements from which they are produced; thus showing them in summary form would not add anything to the diagram.

M-R composition. In the holon diagram the implicit combinations of contextual domains are drawn with dashed rectangles. As we saw in **Fig. 8** (right), the hybridized models represent a mutual constraint that is no longer reducible to the original pair of models. Such emergence (1+1=3) is why contextual analysis is necessary for complex systems. In contrast, the left side of the holon diagram (**Fig. 4**) represents the realized aspects of a system that combine and reduce as discrete entities. Their combinations are such that analysis commutes with synthesis, which is why, as Rosen wrote, their exclusive analysis is too mathematically "impoverished" to represent the system's overall complexity. Nevertheless, their association with each other as a result of sharing a common context may allow them to combine materially without disrupting M-R closure, perhaps as in the formation of organs. Other entailments deciding such combinations are not part of the M-R analysis and thus are not indicated in the diagram, except to show this possibility by grouping them together.

We thus analyze the complexity of the M-R relation avoiding the introduction of new components. The conditions of M-R closure are met by pairing the contextual domains of the M-R components (**<u>A</u>, <u>F</u>, <u>Φ</u>, <u>B</u>**) to define the two circular hierarchical compositions needed to make up the M-R system as a whole, and thus to account for external realization and internal closure. If we further assume that the contextual domain for component **A,** which represents externally produced resources, is always associated with external realization, then there are only three ways of combining the four M-R components into two groups retaining the efficient/material maps specified by Rosen. Thus it is implied that there are three fundamental kinds of M-R systems. Holon **A** is involved with the phenotype since it refers to environmental resources from which the system is produced (via metabolism and repair). It is the only component of Rosen's diagram that is produced by the environment, not from entailments within the M-R system itself. As such, its context must be associated with the realized domain of the environment. Since the three ways of organizing the contextual entailments of the M-R system require that **<u>F</u>**, **<u>Φ</u>**, and **<u>B</u>** contextual models pair alternatively with the **<u>A</u>** model, we may propose that the three possible cell or organism types are characterized by which of these three internally produced holons is associated with external realization. The contextual organization in **Fig. 11** corresponds with **Fig. 10,** where the holon responsible for metabolism, **F**, has been paired with the internalized representation of **A**, and the internal context is formed from the **Φ** and **B** holons. Presumably the opportunities available to such a system for behavior and adaptation would be colored by the characteristics of the holon associated with realization, in this case, suggesting a resource strategy centered on metabolic requirements, a survival strategy that seems most characteristic of **Eukaryota**. The alternative diagrams can easily be drawn in the same manner for the other two possible combinations of contextual domains, resulting in the associations in **Table 2**.

Table 2: Three Cell Types Based on Contextual Oranization

Functional M-R Type	**Contextual Organization**	**Possible Association**
Reparative	FB/ΦA	**Archaea**
Replicative	FΦ/BA	**Bacteria**

Metabolic	ΦB/FA	**Eukaryota**

Of the three possible diagrams the one shown for **Eukaryota** is perhaps the most interesting because it alone unites the genetic functions within a common context, perhaps allowing those functions to become realized in a nucleus. The other two types require that either repair or replication (involving either application or production of **Φ**, respectively) be associated with the external realization context, thus preventing their co-location within a single sub-system. Similarly, since metabolism is more proximally associated with the externally realized domain of the environment, the argument might be made that it is more easily replaced by an externally produced surrogate. Mitochondria, which are the metabolic sub-systems of advanced eukaryotic cells, are now generally believed to have arrived as an invader. **Eukaryota** may thus be expected to have had both primitive and advanced forms based on the degree of compartmentalization that has occurred, aside from but aided by its basic organization.

Rosen was careful to state that his M-R system entailment diagram for life was a necessary condition, not a sufficient one; that there may be other functions involved (as there most certainly are in any currently existing organism). Holon analysis can indicate the logical existence of a system, or sub-system, but various other constraints may be involved in determining how it is closed. Aside from accounting for diversity, this explanation may also help us understand how some components are present only as a systemic quality of the organism, such as the internal model for winter hardening in trees (Rosen and Kineman 2004); or alternatively how a functional need may develop into a realized and identifiable material component, such as an organ.

5. Holon Properties and Clarification of Terms

1. Holons define the fundamental unit of analysis in relational theory. They are infinitely composable and infinitely decomposable.
2. A system is uniquely identified by its identity holon, with four quadrants representing each of the four causes.
3. Whereas a morphism uniquely defines a domain and co-domain, a holon uniquely defines complementary (natural inverse) categories related by structors (as defined here) and functors. The holon is thus a natural modeling relation.
4. An efficient mapping is an abstraction of properties from an ontological natural system; it is not a direct relation between pre-defined sets. Instead, a contextual system provides the order (mathematical structure) that is preserved between sets. Similarly, a final mapping is an abstraction of functions (that express potentials) from a contextual system. It is also not a direct relation between pre-defined functions and in this case a natural system provides the order that is preserved between functions.
5. Hierarchical compositions of efficient/material maps can occur in a system only when final and formal causes intervene between each efficient map in the hierarchy. All compositions must, at least implicitly, involve complete holon compositions (first-order closures), which are hierarchical pairs of realized and contextualized entailments. Every mapping thus has a natural inverse complement somewhere in the universe.

6. Holons have both self-defining and interactive relations, where causes may be shared between holons in any quadrant. Implicitly each bifurcation implies another holon.
7. Wholeness may be defined as hierarchical circularity of all four causes, regardless of where the causes occur or how they are labeled. Wholeness establishes an implicit identity that may accordingly be given a unique set of four holon labels summarizing its causal elements.
8. Holons relate the ontology and epistemology of a system. The system and its context are the ontological elements of nature. Structure and function (which are the codings between naturally inverse complementary ontologies) are the epistemological elements.
9. For every structure there is a system-dependent generating function (efficient map) and for every function there is a context-dependent generating structure (final map).
10. These rules mean that there are no whole systems that exist entirely within the realized domain or entirely within the contextual domain. Each mapping begins or ends at the boundary between realized and contextual systems where we find knowable structure and function; which therefore reference all four causes of the holon. 'Functional entailment' (Louie 2009 pg. 119) implicitly includes all causes.

These properties also allow clarification of the following terms:

- Anticipation: The use of final cause selections within an adaptive system; that is, selection and application of behaviors that best sustain functions of the system through its exemplars (structure).
- Circular causation: The four-part hierarchy of causes that define a holon, including circular hierarchical compositions thereof.
- Closure: Replacement of any of the causalities of one holon (the arrow in any quadrant of a holon diagram) with a corresponding causality of another holon, thus exchanging causes between them and producing a closed path of causation through both holons.
- Complexity: The fundamental condition of nature in which contexts of a system commute inversely but also incompletely with their realizations.
- Context or Contextual System: Systemic conditions, or ambiance (cf. Rosen), in which the potential existence of a natural system or event is established as a natural model. It can be described as a non-spatiotemporal variable (niche or phase) space where conditions for existence are specified independently of the dynamic processes that might realize those conditions. Context is itself realized as environment, iteratively with the events it conditions.
- Entailment: Causation (realized or contextual): The effect of either an efficient cause as a mapping of a natural system (realized domain) to objects of a co-domain (structures); or the effect of a final cause as a mapping of a contextual system (contextual domain) to objects of a contextual co-domain (functions). For example, given a function and natural system it is applied to, entailment of an abstracted structure (property) means that the structure is necessitated by the system as a consequence of the function; or, given a structure and a context it is placed in, entailment of a function is necessitated by that context as a consequence of the structure. Entailments combine two causes and thus are of two kinds, realized

(efficient/material), and contextual (final/formal). Change (functional or structural) is the difference between recursive entailments.

- Function: Expression of systemic potential from a contextual domain (the natural instance of a model), specifying the realization of systems (and, by recursion, their change). Function is the result of formal cause and corresponds with ‘decoding’ (realization) in modeling relations.
- Living system: Minimally, an M-R system.
- Mechanism: Perfect commutation (completeness) of a modeling relation: A natural system or sub-system that can be said to commute exactly with a fully defined system context (singular, largest, computable formal system). The mechanical aspects that can be defined of a system correspond with its measurable and predicable system properties.
- Organization: Entailment and relation in a natural system.
- Relation: Encoding or decoding (mathematically, structors and functors) between realized and contextual systems. Relational coding involves two causes and thus is of two kinds. Encoding is formal/efficient cause, and decoding is material/final cause. Encoding builds models of natural systems as potentials in a context. Decoding realizes models in a measurement space in which events can be said to occur. Contextual relation is the natural instance of a modeling relation.
- Structure: Interactive or measurable properties abstracted from a natural system with respect to (and by) specific contexts. Structure is the result of material cause and it corresponds to ‘encoding’ in a modeling relation.

6. Conclusion

This work has attempted to provide a synthesis of relational theory involving mathematics, philosophy, and natural science. Among these considerations the primary one has been natural science, to which both mathematics and philosophy must defer. Mathematics is probably not the only “language of nature” (Louie 2009, pg. 99), which, like the heart, “has reasons that reason knows not of” (Blaise Pascal). It is arguable where numbers come from; nature responds more directly to “amounts” (Bateson, 1979). Mathematics is a language of thinking beings, most probably attempting to describe the indescribable with greater precision than it contains. It is truly high art and supreme scientific expression, as Louie wrote, but we are also well to heed his admonitions against absolute “mathematical truth” (Louie 2009, pg. 6-7), remembering its subservience to direct experience. Nevertheless, mathematical expression gives rigor and testability to our ideas, and provides a means for analysis, albeit with a tradeoff between particulate and systemic views. Relational theory provides the framework for coupling these views.

Understanding complex phenomena requires that we study the recursive hierarchy of all four causes, which also translate into four components of information and knowledge. To date most of science has been satisfied with using two levels of explanation plus some variation in the third. Not only was that limited by leaving out most of the higher causes, but it prevented knowing about their natural relation with each other. Efficient and material causes, coming from direct measurement or direct inference from behavior, are appealing because they are tangible, as was originally needed to define and incubate science. For a while, the wish for syntactic completeness of just those causes became a trademark of science, a fond hope that many still have. But that era is over now; we have to admit it and proceed. The higher causes are harder to work with because they deal with overlapping, non-discrete phenomena, thus making quantification relative. In the world of context, two systems can sum to one, two, or three depending on how they are organized; but disturbing as that may be, it helps us understand nature.

Relational theory is especially needed today to study changing ecological relations. The niche concept has great significance, as shown here and it has figured centrally in ecological philosophy; but it has never had a clear theoretical foundation. The reason is obvious: the niche is the contextual domain of causes that were considered epistemologically unacceptable in reductionistic science. And yet the niche relation conditions the origin and existence of realized systems. These potentials for existence and adaptation have clear long-term evolutionary effect, meaning that evolutionary pathways are certainly controlled by both realized and contextual domains; by actual conditions and imaged (if not imagined) outcomes. The effect of contextual causes must be reckoned with, as in Baldwin's "new factor" in evolution proposed over 100 years ago (perhaps simultaneously by Morgan and Osborn); or the more recent description of "Niche constructing phenotypes" by F. John Odling-Smee (Odling-Smee 2003).

Nevertheless, much of science is a search for mechanisms and their models. While this synthesis clearly distinguishes a mechanistic description from a complex description, it does not argue that mechanistic science is wrong or inappropriate. It argues that mechanistic science is incomplete and that it can be made much more complete by reconsidering the framework of causal relations in which apparent mechanisms are embedded. The view that emerges is one that is capable of describing both mechanistic and complex aspects of nature, through contextual relations. Although we cannot presume that the answer is final, relational theory does make the claim that its analysis is more thorough and meets all important epistemological criteria for good science. Mechanistic science, having been done well, has successfully discovered its limits. As when Einstein reasoned from paradox in classical physics to a more integral view of space-time, we must now reason from the general mechanistic paradox that excludes system origins, to a broader view that allows us to analyze nature as a self-originating system. Nature appears in this view to be founded on a principle of self-similarity, not by predictable clockwork precision as once imagined. In this new view there is ample room for exploring system origins and nature's internal realities.

Meanwhile, in this synthesis, Rosen's comprehensive works on the subject of relational biology seem to hold up very well indeed, and there are clear directions for further development along lines he indicated and somewhat intentionally left incomplete. Louie's clarification of how category theory applies to relational biology, a particular impetus for this synthesis, was also an important step in completing the mathematical foundation of relational theory and revealing certain incomplete elements.

Philosophers of science have debated the proper epistemological tests needed to justify introducing new theoretical foundations, which have variously been referred to as new paradigms, world-views, theory structures, terms of reference, scientific foundations, or, more simply, our basic assumptions about nature. In the philosophy of science there seem to be six major criteria. Each of these criteria has been argued in this synthesis to the conclusion that the relational approach should be an acceptable paradigm of science, but for the test of time. These criteria, and how they have been met, are listed below:

Necessity	Logically and mathematically necessary to resolve paradoxical aspects of the mechanistic view revealed by the existence of closed loops of entailment. Scientific and social necessity for understanding and managing complex systems.
Consistency	Non-contradictory of the mechanistic view, incorporating and transcending it by placing it into a more comprehensive ontological framework.
Generality:	Generally applicable to all sciences and all natural systems, including cognitive and subtle systems.
Parsimony	Simplest known organization of causes that can explain complexity, mechanism, and life.
Formality	Formalized mathematically for describing and analyzing relations using category theory algebra and holon diagrams.
Fruitfulness	Predictive of organizational patterns that can be tested empirically (e.g., three basic kinds of life). However, meeting this criterion beyond such initial demonstrations requires time and extensive application of the theory in many fields and situations.

The final test is neither scientific nor epistemological, it is social and psychological. The ideas presented here represent a logical step in thinking about our natural world that has had many precursors in the literature and in experimental science. But it is nevertheless a step that is awaiting cultural permission. As we individually and collectively give that permission, deeper insights into our own embedded four-cause nature will begin to emerge. As we realize that nature has its own information relations that underlie the events we observe, we may begin to define our own information systems differently, along these natural principles and thus in a more comprehensive way that admits a much fuller range of nature into our consideration.

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