

# **USING SYSTEMS THINKING AND SOCIAL NETWORK THEORY TO IMPROVE CHILDREN'S MATHEMATICAL PROBLEM SOLVING SKILLS**

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## **ABSTRACT**

The education of young people with mastery of appropriate mathematical skills is crucial to the future prosperity of every country. The gap between rich and poor countries will get wider if young people in underdeveloped countries continue to get a poor mathematical education. This paper presents the initial stages of a systemic effort to improve the mathematical education of young people in a developing nation. Kids, teachers, parents and researchers from quite different socio-economic backgrounds form part of a collaborative learning effort that integrates them using information technology in order to work together to improve their mathematical problem solving skills. Systems methodologies, social network theory, mathematical tests, and qualitative analysis are used to explore how to improve the students' beliefs and attitudes towards mathematical problem solving, their collaborative work, and their mathematical skills. In this project we are making a difference in the lives of young people by taking advantage of their different socio-economic backgrounds, the different contexts in which they live, and their different languages.

Keywords: Systems thinking, collaborative networks, cooperation, learning, mathematics, problem solving, social network theory.

## **INTRODUCTION**

Young people who live in the world's economically most developed countries have consistently obtained much better results in the international mathematic tests that have been done during the last two decades. For instance, this has been the case in the recent PISA and TIMSS surveys. The results obtained by Colombia's children in the 2006 PISA survey reveal that their math, science and reading competencies are much worse than the equivalent competencies of children living in the world's economically most developed countries (OECD, 2007). In the 2006 math PISA tests, Colombia ranked 53 among 57 countries and in the 1995 TIMSS tests, Colombia ranked next to last among 42 countries.

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The above serves as a background to the project described in the following pages. The present article describes a case of the structuring of a social network of collaborative learning in order to improve the problem solving skills of school students of eight schools in Bogotá. It also analyzes how the structure of the social network that we contributed to integrate affects the beliefs and the attitudes of the students regarding the resolution of math problems, the collaborative work and the use of Internet.

### THEORETICAL BACKGROUND

Because the objective of the article is to analyze how the structuring of a virtual network of learning influences in the improvement of the mathematical problems solving skills and in the beliefs and attitudes of students toward the resolution of mathematical problems, the collaborative work and the use of Internet, its necessary to define some concepts related to these issues.

#### Mathematical Problem Solving

According to the Programme for International Student Assessment (PISA) of the OECD, the resolution of problems refers to:

*“... an individual’s capacity to use cognitive processes to confront and resolve real, cross-disciplinary situations where the solution path is not immediately obvious and where the literacy domains or curricular areas that might be applicable are not within a single domain of mathematics, science or reading”.* (OECD, 2004, p. 26).

In order to generate the problem solving skills, its necessary for the individual “... *understand problems situated in novel and cross-curricular settings, to identify relevant information or constraints, to represent possible alternatives or solution paths, to develop solution strategies, and to solve problems and communicate the solutions*” (OECD, 2004, p.3)

A review of literature specialized in this subject allows to identify a number of factors that helps to delimit what we will understand as ‘problems resolution’. These factors include: the existence of a knowledge base; the existence of strategies and the possibility of developing skills to apply them; the role of the control and the supervising that the individuals develop in the attempt to solve a problem; the influence of personal beliefs and attitudes and its regulation and influence on the will of facing and solving a problem; and the use of specific cognitive practices. (see Dossey et al., 2006; Bransford et al., 1999; Mayer, 1985, 1992).

According to Dossey et al. (2006) the resolution of problems includes skills such as problem comprehension, the characterization of problems, its representation, search of solution, reflection on the problem and communication of the problem’s solution. According to Schoenfeld (1985) to solve problems is necessary to use the resources (prior knowledge, mathematical procedures), the heuristics (understanding the problem, defining the strategy, solving the problem, communicating the solution), the control (a constant questioning of whether or not the right direction is being followed and to take decisions accordingly) and the beliefs (affective, attitudinal and emotional resources) (Polya, 1945). According Charles et al. (1987) a fundamental part of the resolution of mathematical problems is the learning of strategies such as drawing figures, solving a simpler problem, making a table, looking for a pattern, making a model of the problem, working backwards.

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In order to acquire these cognitive processes is necessary for the resolution of problems to be a regular and frequent issue in a learning program, thus the student will acknowledge its importance (Charles et al., 1987).

### **Collaborative learning**

In the previous decades, regarding the subject of how the people learn, the researchers have suggested that learning is a social process and that the activities of collaborative learning are essential for students to build their own knowledge (Artzt & Newman, 1997). The interactions between peers enhance the learning because they generate mechanisms such as conflict resolution through disagreements, internalization of explications provided by others, reflexive explication effect because the own understanding is crystallized in the process of explaining to others, positive and negative feedback in the discussions that take place inside the group (Arbaugh & Benbunan-Fich, 2006).

Mathematics learning isn't strange to social interaction. A big part of mathematics learning is achieved through communication processes in social contexts (Forman, 2003). "Opportunities for students to think about mathematics are often associated with their talking about mathematics with one another and with their teachers" (Silver & Smith, 2002, p.63). Participating in social activities, the students have the opportunity not only of learning mathematical skills and procedures, but also they will be able to explain and justify their own thoughts, discuss their observations and observe models of how to use mathematics efficiently in different problems resolution situations. (Hurme & Järvelä, 2005).

As a consequence, it is necessary to establish how to guarantee the collaborative learning. This type of learning refers to the one carried out in small groups of students that work together as a group to resolve a problem, finish a task or achieve a common objective. Some aspects to be taken in account in collaborative learning are:

- Group size: the smallest group possible so every member will be needed and the largest possible so there will be diversity of ideas and skills. The researchers have proposed 3 to 6 students per group in presential groups and up to 15 students in virtual groups. (Bordogna & Albano, 2001).
- Time spent together: in order to generate cohesion the group members must share time (Artzt & Newman, 1997).
- The heterogeneity: the groups must be integrated by students with different skills and socio-demographic characteristics (Graham, 2002).
- Communication skills, peer pressure, reciprocity and individual responsibility (Johnson & Johnson, 1999).
- The establishment of regular routines in order to generate cohesion (Graham, 2002).

### **Computer supported**

Learning can be potentiated with different tools. One possibility is to potentiate it with technological networks. In the virtual environments there is the alternative of synchronous or asynchronous systems. The former tend to acquire more consensus and the latter generate more profound and creative analyses (Benbunan-Fich & Hiltz, 1999). Additionally, the design of the virtual environments must be simple, not overloaded with visual images (Hwang et al., 2006).

The use of the computer to support the collaborative learning has some advantages, for example the information is available at anytime and allows participation equity (Graham, 2002), it promotes an open, safe and reliable learning environment that allows equal opportunities for participating regardless of knowledge levels and without the feeling of being ridiculed or scorned (Dewiyanti et al., 2007).

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Nevertheless, it also has some disadvantages such as the increase of the anxiety caused by the task in hand, due to low frequency of participation of the other members and the delays in the replies; and the possibility of free-riding (Benbunan-Fich & Hiltz, 1999). In the case of mathematics learning there is another drawback: it's possible that the infrastructure that supports the collaborative learning doesn't support the use of symbolic and graphic language (Hurme & Järvelä, 2005; Hwang et al., 2006).

### **Beliefs, attitudes, and incentives**

The beliefs and attitudes that the apprentices have regarding the collaborative learning, the use and the ease of use of the technology as support to this learning (Arbaugh & Benbunan-Fich, 2006), and the development of skills to solve mathematical problems (Schoenfeld, 1985) influence the learning effectiveness. Also, the design of incentives influences the effectiveness of the collaborative learning (Artzt & Newman, 1997; Benbunan-Fich & Hiltz, 1999).

## **METHODOLOGY**

The methodology proposed for the analysis of collaborative learning using virtual networks in order to improve the mathematical problems solving skills includes a structural analysis (social network analysis) of the interactions generated in the network and a qualitative analysis of the message contained in the aforementioned interactions.

### **Social Network Analysis**

The analysis of social networks helps to explore the world of the actors with the resulting social structures derived from the relations established by the actors. In this analysis, the general structure of the network, its groups and the position of the individuals in it serves to penetrate into the social structures that lie beneath the flows of knowledge, information, interchanges, power, learning, among others (Sanz, 2003). Besides, studies have determined that the social networks present emergence aspects (characteristics that come to existence in particular contexts) and history (known relations and shared experiences) (Cho et al., 2005).

Previous investigations (Cho et al., 2005; Daradoumis et al., 2006; Finegold & Cooke, 2006; Hurme et al., 2006; Cho et al., 2007) have studied the influence of certain behavioral patterns of the individuals in the social network related to their performance in learning, attitudes, among others.

The indicators that illustrate this influence can be centrality or cohesion indexes. The centrality indexes are, among others: grade (shows who is more acknowledged by the other actors); betweenness (shows who is playing a linking or intermediary role). The cohesion indexes are, among others: density (shows the evolution of the network's complexity), geodesic distance (shows the evolution of the minimal distance between nodes), reciprocity (shows the percentage of participant interactions that were reciprocal) and the coefficient of clustering (shows the subgroups and the network's fragility) (Wasserman & Faust, 1994; Newman, 2003; Sanz, 2003).

### **Qualitative analysis**

Considering that the communication and argumentation issues are now being regarded as central in mathematics learning (Forman, 2003), the qualitative analysis consisted in a review of the nature of the interactions and agreements of the groups (Hurme & Järvelä,

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2005), as well as a description of the participation and frequency of interaction of the individuals in the network (Finegold & Cooke, 2006).

As in previous researches (Finegold & Cooke, 2006), an analysis of the results of pilot surveys about beliefs on mathematical problems resolution and the teamwork was made. The survey had 30 questions about perception toward mathematical problem solving, 10 questions in relation to teamwork, and 5 questions with regard to use of technology. It was applied to the participants before and after the project.

### STUDY CASE

#### Project description

The Atarraya project consisted in a virtual network of mathematical problems resolution about proportionality. At the beginning, Atarraya was formed by 206 voluntary students of tenth and eleventh grades of seven high schools of Bogotá (5 schools of a low socio-economical stratum and 2 of a high socio-economical stratum), twelve math teachers of these schools and researchers of the Los Andes University Mathematics and Engineering departments. Table 1 shows the socio-demographic characteristics of the participants.

Characteristics	No.
Male	109
Female	97
Colegio 1 (high socio-economical stratum)	73
Colegio 2 (low socio-economical stratum)	17
Colegio 3 (low socio-economical stratum)	17
Colegio 4 (low socio-economical stratum)	18
Colegio 5 (low socio-economical stratum)	23
Colegio 6 (low socio-economical stratum)	14
Colegio 7 (high socio-economical stratum)	44

**Table 1. Characteristics of the participants**

The objective of the network was that the students worked collaboratively in order to solve mathematical problems. For that purpose an application was designed (see figure No. 1) with password restricted access, given to each student of the project. The network was created initially with heterogeneous groups (in terms of schools and gender of their members) of 5 students.

Before beginning the project the students were gathered so they filled out the survey on computer use, teamwork and mathematics learning. Besides, a group dynamic, that required virtual communication, was done. In the end of the project a similar activity was carried out in order to corroborate the possible changes resulting from the process.

The network functioned from September 2006 to November of the same year and from February to May of the following year, with a problem each month (7 problems in total).

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The idea was that the students examined each problem and worked both individually and in group, using the group's forums and chats available for that purpose. Additionally there were general chats and groups (for the interaction of every student of the project) with the purpose of generating a social environment for the entire network. After discussing the resolution of the problem, the students could send a group agreement with the problems solution. The researchers checked the solution and feedback the group, mainly in two ways: encouragement to keep participating in the network and guidelines on some missing or wrong elements in the sent agreement.

At the beginning of the third problem the directive board of one of the schools involved (school No. 7) decided to drop off the project. The decision was made because, at that time, students had many extra class works and activities and needed to concentrate on their projects. On its behalf, the Atarraya project administration decided not to restrict the access to the website to the 44 students of the mentioned school. Therefore, if any student, by personal initiative, wanted to continue in the project, he or she could do it.

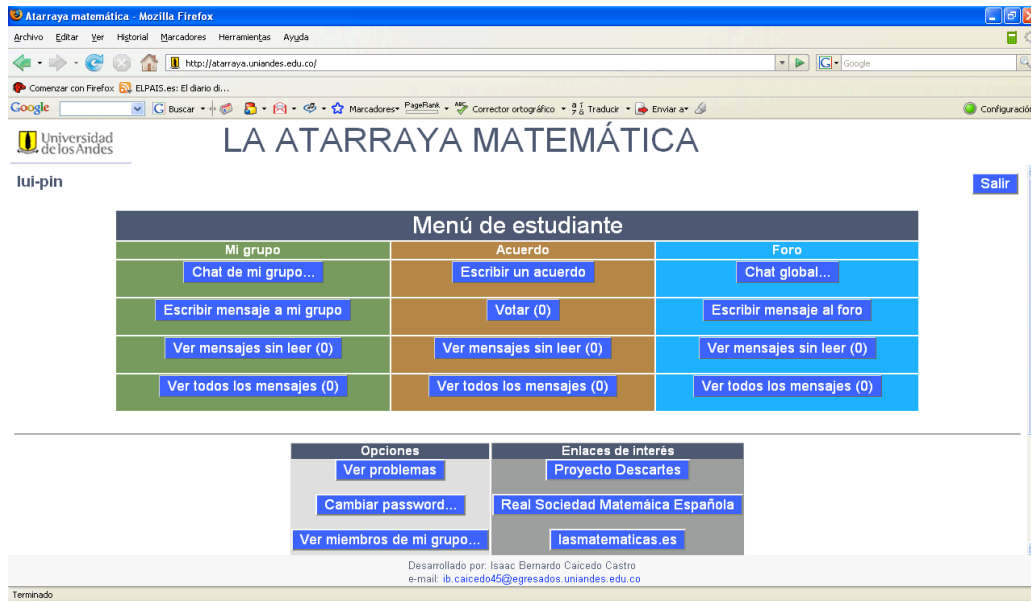
On the other hand, many students expressed their difficulties to get access to the Internet and participate actively in the project. These students were members of the 5 public schools, of a socio-economic low-level, with limited access to computer and Internet at the facilities, because the few computers available are used in classroom activities. This became a determinant factor of the low-rate participation.

During the entire project there were periodic (monthly) gatherings with the 12 teachers and two leader students of each school with the purpose of evaluating the structuring process in terms of the projected goal. In an open to dialogue environment, the participation in the virtual network was analyzed and alternatives for encouraging the participation in the network were proposed. Some of the changes proposed in these meetings were:

- Since the participation was entirely voluntary and didn't generate incentives in the classroom (it wasn't linked to the class curriculum), there were several stimuli designed throughout the project in order to encourage the participation. The incentives changed throughout the project. The most common were: iPods and movie tickets raffled between the groups that discussed the most about the problem and send agreements.
- Because there were 206 students invited to the project, but only near to 25% of them participated actively, the network was restructured in the middle of the project with two type of groups: the groups formed by students with active participation (8 students per group) and the groups formed by students with very little participation (10 students per group).
- Finally, due to the difficulty to access Internet of the low stratum students, a computer room was made available for 3 hours per week in Los Andes University to facilitate their access to the virtual network.

a)

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b)



Figure 1. a) Students menu for the atarraya.uniandes.edu.co website; b) Website for posting messages in the group's forum

### Data

The data gathered, that is the basis of our analysis, is surveys taken by the participants on their attitudes and beliefs regarding the resolution of mathematical problems, the collaborative learning and the use of Internet. Additionally, the interactions of the individuals in the network were examined as well as the content of those interactions. It's important to stress that our unit of analysis is the individual.

### ANALYSIS

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In order to achieve the objective of exploring how the network structure affects the mathematical problems resolution skills and the students' beliefs on this subject, the collaborative work and the use of Internet, two types of analyses were made. The first one consists in a description of the interactions and the agreements developed by the students in one of the 7 problems ("the rice problem"). Linked to that description is the analysis of the surveys on beliefs taken before, during and after the project. The second one consists in a structural analysis of the networks, keeping in mind some cohesion and centrality indexes. Both these analyses lead us to establish a series of correlations between the described variables with the purpose of reflecting on this exploratory study.

### Qualitative analysis

#### Types of participation

Among the 7 problems proposed to the students during the study time, one that had one of the highest number of interactions, and quality of these, was the one we named "the rice problem", in which 7 brands were presented with their respective discounts, and the students had to identify which ones had the same discount and which ones were the most and the least attractive for the buyers. In this problem 42 out of the 206 students inscribed in the program participated, that is, the 20,4% performed some type of interaction.

The time granted for "the rice problem" was a month, during which the students had access to the website and its resources, that is, the forum and the chat (the general one and the group one). During the first and the last week the number of interactions was higher than in the rest of the month; the first case is explained by the novelty, the 42 students connected to read the problem and to the discussion sites (forums and chat) to reach an answer with their teammates. However, as time went by and the answers from the rest of the members didn't arrive the interest decreased until the week in which the agreement had to be posted in the designated website.

Of the 42 people that read the problem, 34 (16,5%) participated actively in the forums (the group ones and the general one). Only 5 groups (of the 24 groups of the network) had members interacting among themselves in the group forum, with an average of 3 members per group interacting (that is, a third part of each group).

Of the interactions, the ones with a higher mathematical content were the ones that took place in the group forums and in the general forum, where each member posted what he or she had accomplished so far, or questions regarding the problem or the group dynamic. Figure No. 2 shows the percentage of communication sorted by the type of interaction that took place in the network forums, from a total of 165 messages.

A social interaction (9%) meant messages to set dates, remind events, among others. A mathematical interaction (49%) meant the discussion about the mathematic issue of the problem (proportionality) and the proposed strategies to solve the problem (representations, relevant data selection, make a model of the problem, among others). An interaction on attitudes (15%) involved commentaries in favor of (or against) the problem, the teamwork or the virtual environment. Finally, we consider "noise" (27%) the messages that didn't generate any of the aforementioned types of interaction and that we regard as trivial. Table 2 shows an example of the type of interaction developed in the forum.

Of the 49% of mathematical interaction, the 14,3% represents interactions about strategies to solve the problem. Because of the low volume of messages regarding solution strategies for

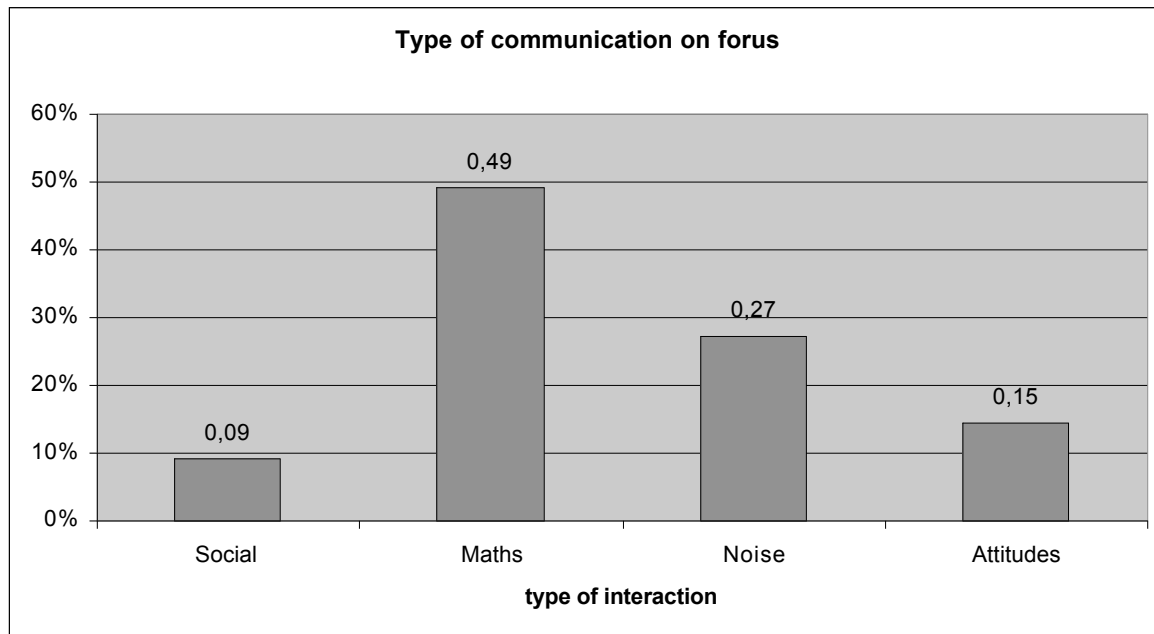


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the problem, the mathematical discussions focused on answering the questions, with little analysis of the procedure used to answer them.

On the other hand, interactions in the chat rooms (both general and group), were social and noise (both categories add up to a 100%). In this case, it seems that the chat rooms generated a social environment in the network.

The 5 groups that managed to interact with an average of 3 members interacting per group, presented agreements with the solution to the problem. However, reviewing those agreements, only one (1) presents the answer in a systematic way, along with the procedures and analyses made to reach the answer.



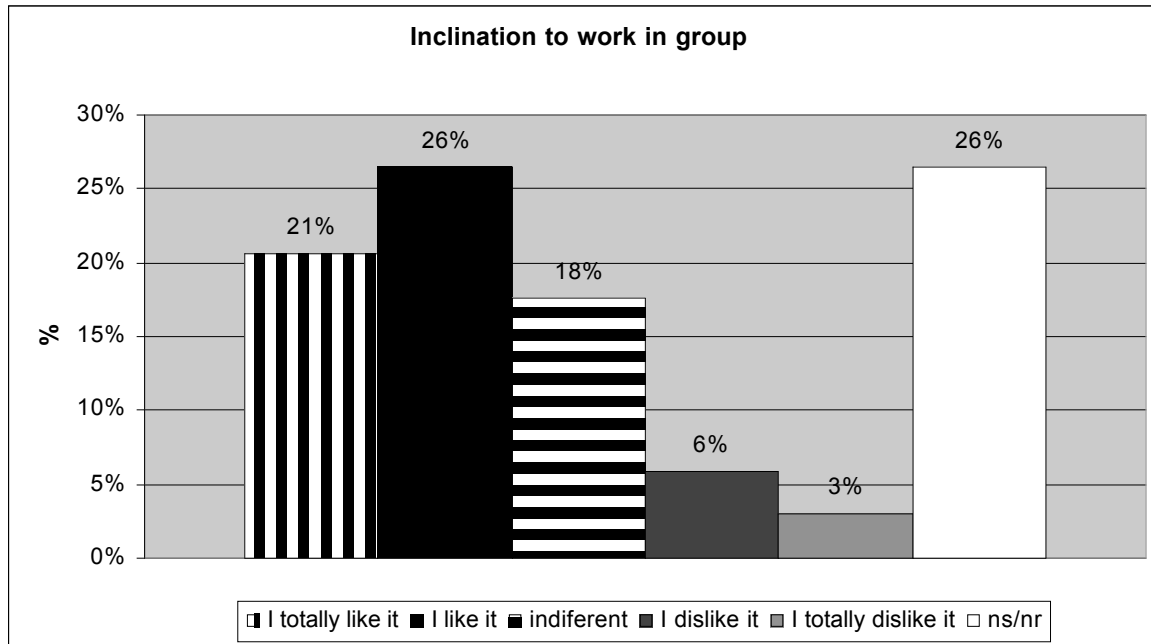
**Figure 2. Types of communication in the forums, for the rice problem.**

With this 16,5% of active students some perceptions regarding teamwork and mathematics were reviewed, before and after the project.

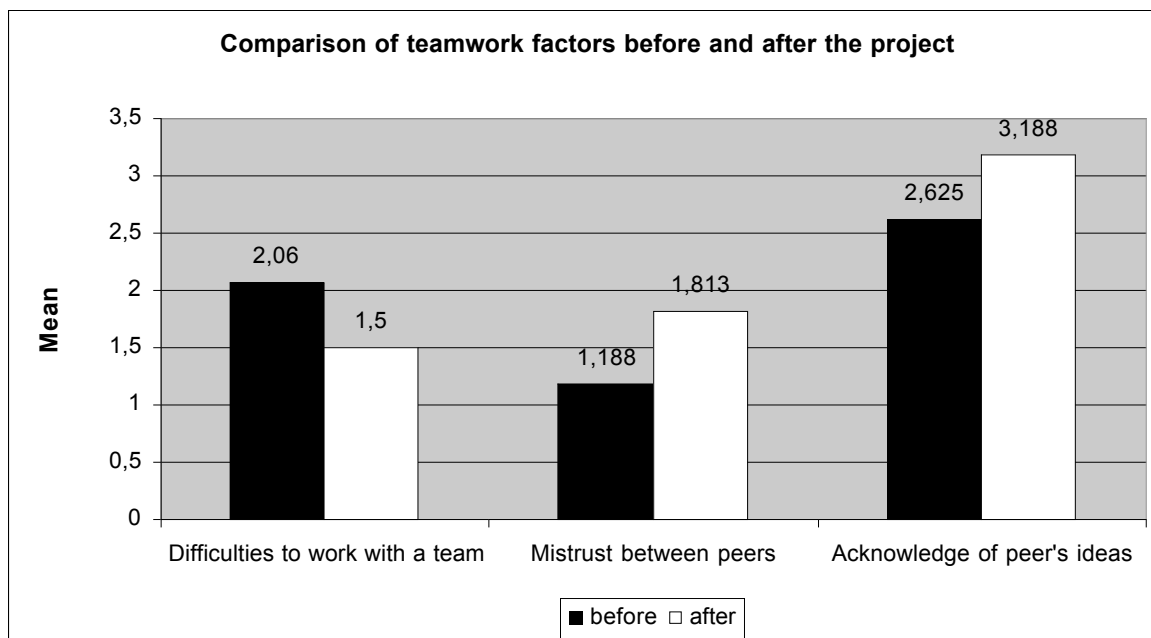
### Teamwork

A very important skill in mathematical problem solving is to work in group. Figure 3 shows that, before initiating the project, 47% of the participant students liked to work in groups.

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**Figure 3. Inclination to work in group.**



**Figure 4. Comparison of teamwork factors before and after the project**

After the group work done during the months of the project, the students evaluated the teamwork and noticed less difficulty in this type of work and more acknowledgement of the ideas of others. However, increase the distrust toward the proposals of others. This can be seen in the Figure 4. This results are statistically significant with an alpha of 0,05 (for the difficulties to work in a team and mistrust between peers cases) and 0,1 (for the acknowledge of peer's ideas case)

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*Ruby, G18, FGrupal:* For the first question. The offers that are very alike are options 1 and 3 because in these two options the price is calculated and the 3<sup>rd</sup> kilo you buy is not paid. Second question . I don't have an accurate answer because the option 6 could be the least appropriate to buy when the rice offers. For question 3 I think it could be the option because the 3<sup>rd</sup> kilo is free...but look that all offers have a big discount and isn't just that, but for all options is given or analyzed the same to know that in all offers the 3<sup>rd</sup> kilo is free doesn't matter the brand of rice chosen .

*Ruby, G18, FGrupal :* have in mind that besides the offers given by the rice brands... the market also gives an additional 15% discount, then if we buy 2 kilos they wouldn't charge us \$2000 pesos but less because they would compute both discounts (the offer's and the market's)

*Ruby, G18, FGrupal:* here I send you another way to solve of the first item.

*Jhon, G18, FGrupal :* analyze this and then respond me: 1: I consider the two rice brands with the same discount are rice tigre and rice casanare, even if the offers are different the final discount is the same. 2: The least appealing offer for Jairo and Juanita is the given by rice La rebaja, because both of them want that when buying rice the best option is the one to save some money, therefore, the discount given by rice La rebaja is not the best , is the least significant within the offers. 3: Arroz del oro 1k=25% 2k=30% 3k=50% 1k= 750(25%) 2k= 700(30%) 3k=500(50%) total=1950 1950-292.5(15%)=1657.5 – This is the best offers for Jairo and Juanita , first of all, it gives a fear discount per kilo, somehow reasonable, and second, because it gives the buyer the opportunity to save some money when buying a certain product.

*Ruby, G18, FGrupal:* Hi, I liked very much the fact that someone of our group finally participated.

*Jhon, G18, FGrupal:* Hi, rub-mor, let me know your answers so we can compare and reach some kind of agreement

*Jhon, G18, FGrupal:* The lack of voting on my agreement is caused by computer failures, for example in mine, I try to download the attachments and they won't open. Regarding my group, I have one connected already. With the calculations I have several doubts: la rebaja offers says " In rice la Rebaja they gave bonus of \$200 pesos, and the publicity said: for the first kilo bought, pay with one bonus, for the second give two, and for any additional kilo give two bonus as part of payment" this means: for 3 k you give 2 bonus of \$200 pesos, or for 2k pay with 2 bonus, and for the 3rd kilo, Which is the additional one (depending on the brand) they give 2 bonus of \$200 pesos, regardless the 2 bonus given for the 2 previous kilos? The other doubt is that it isn't very clear to me the offer of rice el tigre because it says: "Rice tigre offered a discount of 20% on the total of purchase, and on the 2 kilos rice packages it said: Our scale is broken to your favor, our 2 kilos packages are now 2, 5 kilos" because I don't know if the scale is broken giving us extra 0, 5 kilos. The logical thing is to complete de 3 kilos buy other package of 500 gr. (0, 5 kilos), because the 2 kilos package is giving us 1/2 kilo for free. Please respond here or by mail at: joffrygomuz16@hotmail.com

*Ruby, G18, FGrupal:*

*Jhon, G18, FGrupal:* What's up guys? Isn't there any one to help solve the problem? Get wit it!

### **Table 2. Example of the interactions in the group forum.**

The above can be ascribed to several causes. First, during the project the students were more aware of the mathematical issues, thus they were inclined to acknowledge as well as to question the proposals of the other members of the team (Forman, 2003). This kind of interactions generated more acknowledge of peer's ideas and less difficulties to work with a team.

Second, the fact that the virtual interaction became difficult and that some of them were acquainted because of their schools caused that, in general (with member of the network outside of their own school), distrust was generated, while the interaction with members of

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their own school turned out smoothly. This possible cause is united to the fact of the emergence and history of the networks: the existence of pre existent social circles (Cho et al., 2005) affected the creation of new connections and limited the actions between group members; on top of that, the differentiated cultural context broadened gap between the group members.

Third, the participation on the project, as mentioned earlier, was not periodic within the time of each problem, not generating the sufficient cohesion, as no habitual routines were established (Artzt & Newman, 1997; Graham, 2002). As a consequence, more anxiousness was generated because the periods between responds (Benbunan-Fich & Hiltz, 1999). This fact raised the mistrust between peers.

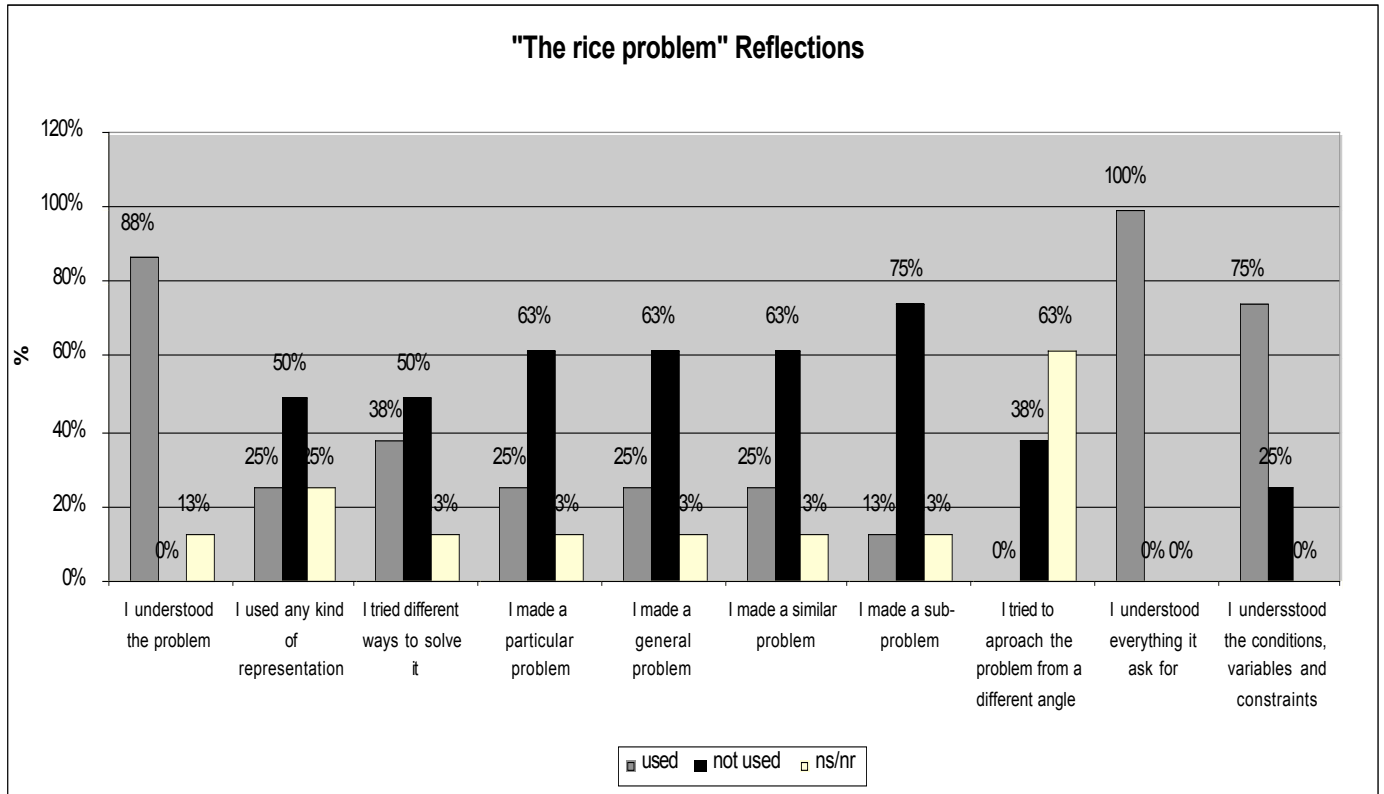
### **Beliefs, attitudes about mathematical problem solving**

Another important aspect in mathematical problem solving is what students believe about this issue (Schoenfeld, 1985). As part of the self evaluation that the students made during the learning process they answered a reflection with their attitudes, beliefs and perceptions about the problem comprehension and the establishment of strategies to solve it. This reflection was made for the rice problem and shows how students themselves evaluate their improvement in the skills in problem solving.

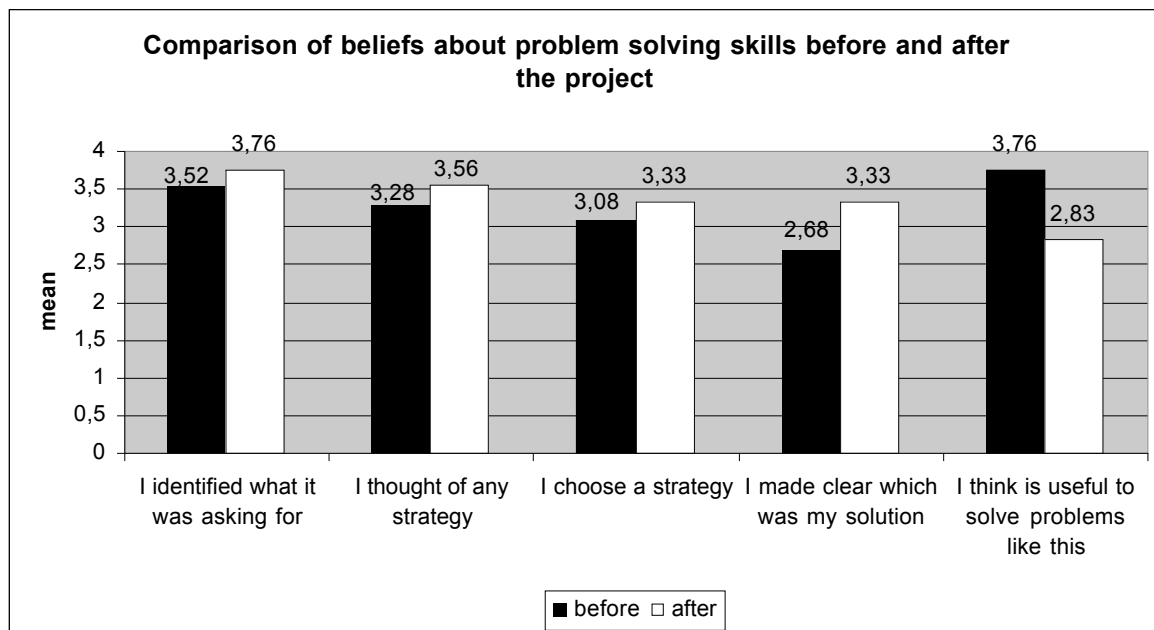
The results of that reflection (shown in the Figure 5) show little employment of problems resolution strategies, in spite of their alleged understanding of the problem.

Figure 6, on the other hand, shows the positive perception that the students have on their initial and final state in problems resolution regarding the comprehension of the problem. In other words, comparing the answers before to and the answers after the project, students believe that they have improved their skills to solve mathematical problems. Moreover, an aspect that draws powerfully the attention is the little utility they find in mathematics and, specially, in solving mathematical problems (see results to: I think is useful to solve problem like this). Despite of the Figure 6 shows differences in beliefs before and after the project, the results of significance test show that only the last question (I think is useful to solve problem like this) is significant with an alpha of 0,1.

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**Figure 5. Reflections on the self evaluation about the learning of problem solving strategies**



**Figure 6. Comparison of personal beliefs about problems solving skills before and after the project**

## Social Networks Analysis

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The 206 students of the project interacted for a period of 7 months (with a problem each month). For each problem the interactions established in the forums (the general one and the group ones) by the participants were considered as a social network. The conversation that took place in the chat rooms weren't taken in consideration because most of them consisted in trivial messages that didn't generate discussions on mathematical topics.

Each one of the 7 networks established by the students was described through the following cohesion measures: density, clustering coefficient, reciprocity, average distance and compactness. The results of those measures are shown in Table 3.

COHESION INDEX							
	network problem 1	network problem 2	network problem 3	network problem 4	network problem 5	network problem 6	network problem 7
Average density	0,0451	0,0004	0,1095	0,0202	0,012	0,0076	0,0101
Standard desviation density	0,3369	0,0232	2,6173	0,2754	0,1555	0,1192	0,1858
Clustering coefficient	0,553	0,189	4,439	1,521	0,846	0,888	1,636
Reciprocity	0,052	0	0,2198	0,1549	0,1299	0,125	0,1747
Average distance (among reachable pairs)	2,169	1,278	2,563	2,003	2,222	1,627	1,941
Compactness (cohesion)	0,072	0	0,07	0,022	0,017	0,008	0,009

**Table 3. Measures (SNA) of the networks established in the project**

**Note: problem 4 is the rice problem.**

From Table 3 it can be inferred that, in general, the networks established by the students were very fragmented (the cohesion-compactness index is very low in all of them). All the measures are consistent among themselves, though is important to notice that, spite of the low cohesion of the network, in some cases such as in the network of the problems 3, 4 and 7, the interactions were slightly more reciprocal.

It must be taken in account that between the problem 3 and the problem 4 there was a change effectuated in the structure of the network, which consisted in the generation of new work groups with the students that had participated actively in the project and, at the same time, the creation of other groups with the inactive students. This change took place since problem 4, and remained until the last problem. Although the change didn't affect the cohesion of the network, it generated more reciprocal interactions. It is possible that the generation of reciprocal interactions was improved by the incentives and personal motivation given within the active student on the network (Arbaugh & Benbunan-Fich, 2006). In addition, several students were classmates as well as project group mates, making the existing networks to be more reciprocal (Cho et al., 2005; Hurme et al., 2006).

Additionally, the behavior of each actor in the established network was examined with the centrality measures: grade (in and out) and betweenness. The results of these measures show the following:

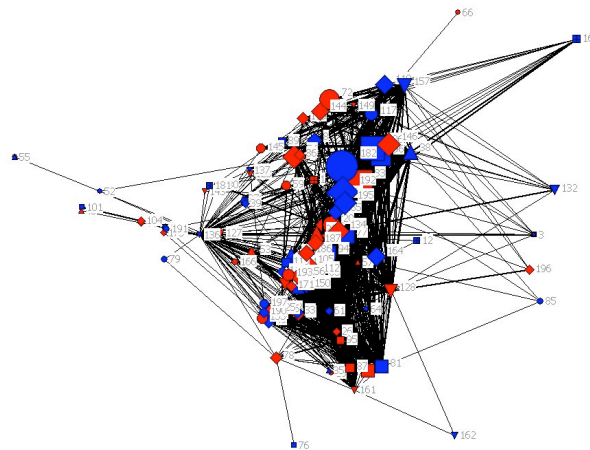
- In the networks generated in the problems 1 y 2, there's no evidence of a distinguishing behavior of the participants. This is the result of high-participation level of the school No.7 in the problem 1, the same that later on drop off the project. Because of that, there are no central actors of that school in the rest of the project. Additionally, in the second problem there were almost no activities on the net.
- In the networks generated in the problems 3, 4, 5, 6 and 7 there are some noticeable behavioral patterns in some participants. Some members became central nodes

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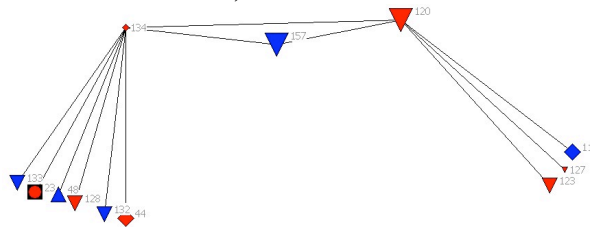
- inside of the network during these last five problems (such as 120, 101, 154, 1 and 2). Other members become intermediaries (such as 181, 38, 172 and 143).
- These members with central positions in the network have 18% of the messages about mathematical discussions.
  - This type of participants generates a network dynamic that provides certain agglomeration and reciprocity.
  - The networks, in general, remain stable in its number of participants, even in those that participate actively.

In the Figure 10 we can observe the interactions that took place in the described networks.

a) Problem 1

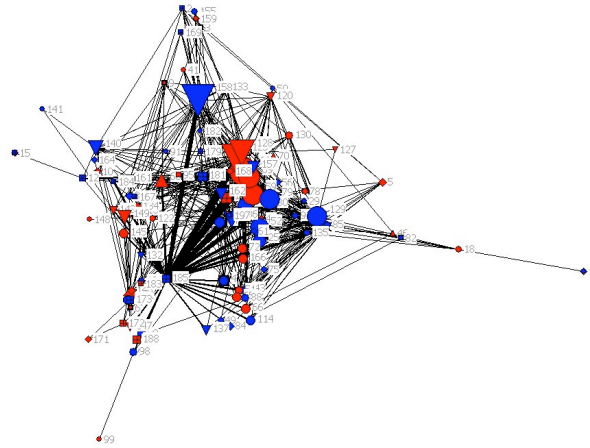


b) Problem 2

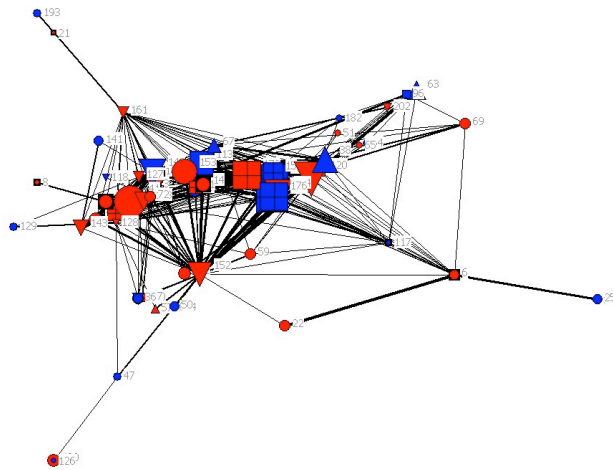


c) Problem 3

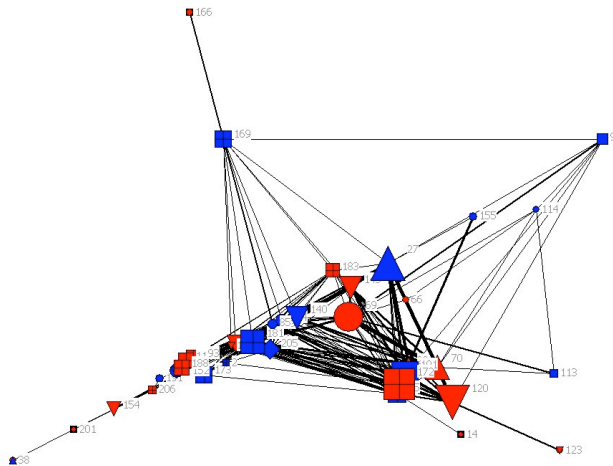
## Systems Thinking and Social Network Theory



d) Problem 4



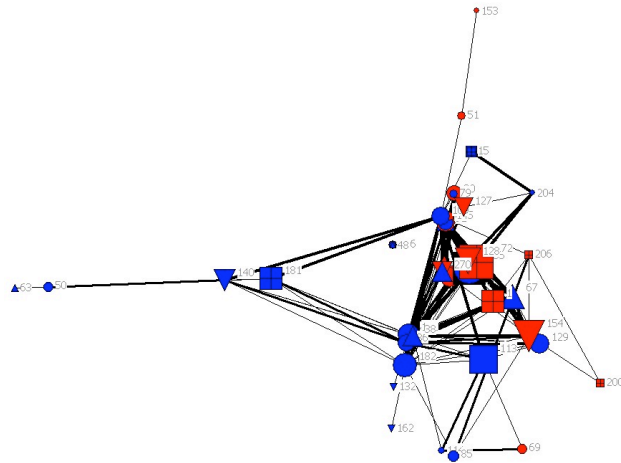
e) Problem 5



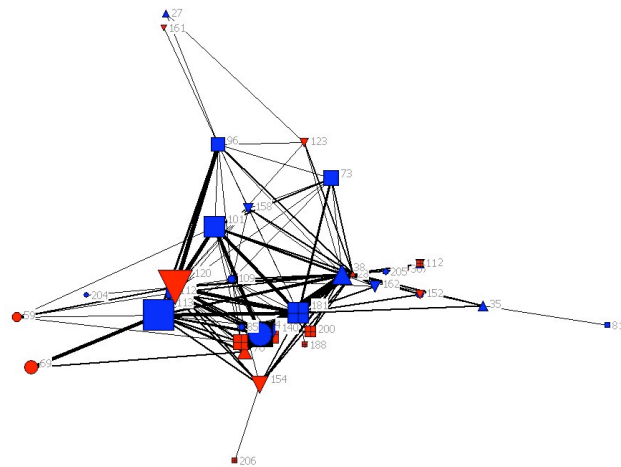
f) Problem 6



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g) Problem 7



Attributes	Convention
Male	Blue
Female	Red
In-degree	Node size
School 1	Circle
School 2	Square
School 3	Upwards triangle
School 4	Box
School 5	Downwards triangle
School 6	Box in circle
School 7	Diamond
Number of interactions between nodes	Arch size

**Figure 10. Interactions during the entire project of collaborative learning.**

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Finally, an analysis of correlations was made between the variables of the survey and indexes of the network that were studied in this project. From that analysis the following can be said:

- There is a positive correlation between those factor affecting the work in team and learning of mathematical problem solving skills (0,63). In other words, the students felt that thanks to team work and knowledge exchange within the network they can improve their own skills (Silver & Smith, 2002).
- There is a positive correlation between the level of trust developed within the teams and the reciprocity of the messaging (0,55), and the clustering (0,52). This means, that each the reciprocity and the clustering affect the level of trust. It also means, that the reciprocal communication between peers was even more rich and of higher quality when existing trust.
- The participant in the network central positions affects positively the reciprocity and clustering (0, 81). In other words, the main actors of the net (regarding in-degree, betweenness and out-degree) are the links over which the important communication processes of the net (mathematical discussions) are structured (clustering and reciprocity) (Hurme et al., 2006).
- After the network's structure change, the central positions and the network's dynamic was related to its history, which is why, starting from problem 4, the network has a positive correlation in terms of participation rate (0,65) and the behavior of the participants in central positions (0,72). This shows the emergency conformation and history of the social networks. In other words, after a certain number of interactions on the social network, there is a strong relation between the prior interactions and the future ones, and the behavior of the participant on the net.
- The participant in the central positions on the net conform the group of student who think in strategies to solve a problem before doing any mathematical operation (0,626).

### FINAL CONSIDERATIONS

The results of this exploratory study show mainly a positive change in the perception of the students towards collaborative learning in mathematical problems solving and their own attitudes about mathematical problem solving. They also show the important role played by the main actors of the network in the production of that type of collaborative learning, where reciprocity is generated and the teamwork structuring is achieved.

On the other hand, it could be seen that, even though there were 206 students invited to participate, only a small fraction (close to 25%) was willing to do so. Because of this low participation rate, the structuring of a learning network wasn't entirely possible.

The above leads to a series of considerations:

- First, regardless the active participation of the student is low, it is necessary to have in mind that the project counted on volunteer students. It is possible then, that if we want to create a new collaborative virtual work network, it should be linked to the student's curriculum, generating even more participation.
- Second, a factor that may have influenced in the motivation of the students to participate was the duration of each problem (a month), that caused that the efforts of the students occurred only in the beginning and in the end of each month (Charles et al., 1987).
- Third, the training of teachers and their continuous accompaniment in process such as the ones described in this article are vital factors for the achievement of the goal proposed in this project.
- Fourth, the difficulty to access Internet of a significant percentage of participants may have also been an important factor of the low participation rate. The lack of appropriate and sufficient equipment within the socio-economic low-level schools was an important variable for the purposes of this work. Although, in virtual learning project more equity

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is achieved (Graham, 2002), it is important to guarantee a minimum of technology requirements to pursue this collaborative virtual learning.

- Fifth, the type of interactions on the net changed depending on the context in which the students interacted in. In the chat the interactions were social. Instead, in the forums the interactions were guided toward the problem solution. This is important, so in the next projects this social process should be considered in the net.

The experience obtained in this project puts forward new challenges related to the obstacles presented in this article. A work derived from the analysis of this project is currently being carried out with elementary school students from one of the participant schools. For this new project, is considered, besides the variables described in this article, the improvement in the problem solving strategies regarding the problem comprehension, solving strategies, process control and communication of the answers (Schoenfeld, 1985). The mentioned project started on October 2007 and in the first two problems developed, it was found an active participation level of 33% within the 75 students of fourth grade of elementary school (a rise near to 8%, comparing it to the project described in this article). This work is taking in account the considerations obtained from this investigation and constitutes the next step in the road to answer the question of how to use collaborative work in a virtual network in order to improve the mathematical problems solving skills.

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