MODELING AND OPTIMIZATION OF LIFE INSURANCE POLICIES

Perla Rocio Calidonio Aguilar, Chunhui Xu
Dept. of Management Information Science
Chiba Institute of Technology
Chiba275-0016, Japan

ABSTRACT

In this paper we try to determine the "Optimal Premium" that should be charged to the policyholder in order to maximize the company's profits on a specific type of policy in order to meet all the contractual obligations of the company due to the minimum guarantee rate and participation rate. In the past, many papers have analyzed the pricing of insurance policies in order to make a "fair" valuation of it; however, we consider that the fair value doesn't represent the real world; therefore will take a more realistic approach from the management point of view.

Typically modern insurance products embed several types of financial options, features that increase their complexity. As a result their accurate valuation is an issue of great concern for life insurance companies, not just because of solvency problems that might arise but also because of competitive pressures. This paper contemplates a life insurance policy, in particular of the endowment type in which a minimum return is guaranteed to the policyholder and according to the performance of a particular investment portfolio during the year, an additional amount may be credited.

Keywords: optimization, policies with profits, fair pricing

1. INTRODUCTION

Life insurance companies can be led to catastrophic situations if they don't pay more careful attention when deciding the premium that will be charged to the policyholder and when carrying out the valuation of the options embedded in it. Furthermore, not just the inaccurate pricing of the policies and options represents a potential hazard for the company but also ineffective management of the company's assets and liability. As a result, these problems have drawn attention of insurance companies, investors and researchers.

In order to gain ground in a very competitive environment, life insurance companies have become more innovative when designing their products. Usually these contracts embed different types of financial options, which make the contract attractive not only for those who seek insurance but also for those who seek profitable investment opportunities. Minimum guarantee rate of return, reversionary bonus, right of surrender are some common features that we can find in life insurance contracts everywhere in the world. However, as much as these products have become innovative, they have also become more complex and thus the proper determination of the premium to be charged is an issue of great concern for life insurance companies.

1.1. With-Profits Policies

One of the most common life insurance products is the so-called participating policies or with-profits policies. In this type of policies, the premium paid by the

policyholder is invested in the life insurance company's with-profits portfolio and the profits are shared with the policyholders. Usually with-profits contracts have a basic amount of money assured, which is determined at the inception of the contract. This sum assured is payable on death or at maturity of the contract, whichever is earlier. Periodically interest is credited to the policy, this interest is linked to the performance of the company's with-profits portfolio, and thus depends very much on the performance of the company's investment approach, market condition, etc.. Generally this interest cannot be less than a predetermined interest rate, it is to say, there is a *minimum interest rate guarantee* to be credited to the policy each year. Whenever the return of the company's with profit portfolio exceeds the minimum interest rate, the policyholder receives a percentage of that excess rate (Bonus). Once that the minimum interest rate guaranteed and bonuses have been credited to the policy account they cannot be taken away. Together the sum assured, guarantee rate and bonuses credited have been one of the main features that attract investors' attention.

The crediting mechanism of the bonuses may take two forms: maturity guarantees where the benefit is paid at maturity of the contract and multi-period guarantees where the benefit is gradually distributed at the end of a determined period of time. In the case of maturity guarantees any excess return in previous periods can be used to build up a reserve for bad times, however in the case of multi-period guarantees any excess return made in early period cannot be used in bad periods.

As in the case of policies with a minimum rate of return guarantee, guaranteed cash values may generate significant risks for the life insurance company.

Even though with-profits were not considered so risky investments, nowadays companies are not certain if they will be able meet their obligations and, as it has been shown recently; several of them have failed to do so. One possible cause of this event might be that the insurance companies did not comprehend the complexity of these products due to the different options embedded in it, and thus neither understood how the market movements will affect their return. During the 1990s markets rates dropped and as a consequence the rates of return of the life insurance companies' portfolios have lowered. Before the market rates collapsed insurance companies issued policies, which minimum rate guarantee seemed to be out of the money but as market rates started to become lower, the interest rate guarantee started to be in the money, leading insurance companies to incur in significant shortfalls. Hence, several companies started cutting payouts in order to deal with shortfalls and prevent bankruptcy other have defaulted due to have been unable to meet their obligation, not just to their policyholders but also to shareholders.

An example that can illustrate this situation is the insurance giant Norwich Union, which is part of Aviva group, the world's seventh largest insurance company and the biggest in the United Kingdom. When in the last decades the mis-selling of endowment policies was widespread, Norwich Union and many others insurance companies stopped selling with-profits endowment policies. However, considering that as one of the biggest providers of these products and the fact that typically the maturity of these products is long time after they started, Norwich Union still has a lot of customers waiting for the maturity of the contract and wondering how much will be worth the policy by that time. By the beginning of 2007 it was revealed by the company that nine out of ten with-profits endowment polices will probably fail to fall

short on their target.

1.2. Current Models

Early work on this direction is found in the work by Brennan and Schwartz (1976) and Boyle and Schwartz (1977) where they contemplate the fair pricing of equity-linked policies with maturity guarantees. More recent contributions addressing *fair* participating policies with guarantees were considered by Miltersen and Persson (1999), Miltersen and Persson (2003) Bacinello (2001), Consiglio, Cocco and Zenios (2001). Regarding the fair pricing of participating policies with guarantee and surrender options we found the works by Grosen and Jorgensen (2000), Miltersen and Persson (2000), Miltersen and Hansen (2002) and Baccinello (2003).

Grosen and Jorgenses (2000) show that a typical participating policy can be decomposed in three elements: a risk free bond, a bonus option, and a surrender option. Then they construct a dynamic model and value these three elements separately using contingent claims analysis. In their work, they use Monte Carlo Simulation for participating European contract, which consists of a risk fee bond and the bonus Option. Furthermore they use the recursive binomial method in order to price the American participating contract, which comprehends the 3 elements mentioned above.

Bacinello (2001) analyzes the *fair pricing* of a life insurance endowment policy with a minimum return guarantee and participation rate. In her approach, Bacinello considers mortality risk and financial elements; furthermore, she evaluates premiums paid either by a single amount at the initiation of the contract or by periodical premiums. Using Black-Scholes model for the evolution of the reference portfolio and martingale theory, Bacinello derives a closed-form relation for fair contracts.

Miltersen and Person (2003) evaluates minimum rate of return guarantees in connection with a distribution method for excess returns. In their approach, they consider a policyholder account,   where a minimum guarantee rate and a percentage of the surplus of the benchmark portfolio is credited at the end of each year. They also consider a bonus account which is credited in years when there is positive difference between the realized rate of return on the benchmark portfolio and the minimum rate guaranteed and; conversely, when this difference is negative, funds in this account are reassigned to the policyholder's account in order to meet the minimum rate of return guarantee.

Respecting to work done into assets and liability management for participating policies with guarantees Consiglio, Cocco and Zenios (2001) develop a scenario optimization asset a liability management model for multi-period participating policies with guarantees. In particular, in their model the assets' structure is endogenized, in contrast with the literature mentioned above, where they consider an exogenous structure.

Other examples of the use of portfolio optimization for assets and liabilities management includes the Russel-Yasuda Kasai Model formulated by Cariño and Ziemba (1998), The Towers Perrin model of Mulvey and Thorlacius (1998), the CALM model of Consigli and Dempster(1998). These models have been used

successfully in a practical setting but their application does not cover participating policies with minimum interest rate guarantee.

In this paper is consider a participating life insurance policy with a minimum interest rate guaranteed and surrender option. The target is to build a model for the valuation of the insurance policy described above and propose a methodology in order to decide the most proper premium that will maximize the return of the policy in order to meet all the contractual obligations of the company due to the minimum guarantee rate, participation rate, etc…

The valuation of life insurance policies is influenced by many factors; it seems that a model incorporating all these factors would be somewhat complex. Thus the model is intended to incorporate the imperative factors and at the same time keep it tractable.

As outlined before, many papers have analyzed these policies in order to make a "fair" valuation; however, in this paper we will take a more realistic approach from the management point of view and try to determine a reasonable premium that should be charged to the policyholder in order to maximize the policy's profits. Furthermore, we make use of the approach proposed by Baccinello(2001) in order describe the expected payments (fair price of the contract). In order to find the optimal premiums we solve the model using the Soft Approach method put forward by Xu (2006).

The paper is organized as fallows: Section 2 describes the evaluation of the life insurance policy .In Section 3 we present the methodology to determine the profit. The formulation of the optimization model is addresses in Section 4. Numerical results are shown in Section 5 , and Section 6 concludes the paper.

2. EVALUATION OF THE LIFE INSURANCE CONTRACT

In this section we describe the assumptions and methodology proposed by Bacinello for the fair valuation of the policy. Following previous literature, financial markets are assumed to be perfectly competitive, frictionless, and free of arbitrage opportunities. Moreover, agents are assumed to be rational and no satiated, sharing the same information.

The continuously compounded market rate is assumed to be deterministic and constant. Consequently, a stochastic evolution followed by the rates of return on the reference portfolio is the source of the financial risk; furthermore, the evolution of the rate of return on the reference portfolio is assumed to follow a geometric Brownian motion:

$$\frac{dS_t}{S_t} = rdt + \sigma \, dW_t \tag{1}$$

Here σ is constant and represents the volatility parameter and W_t is a standard Wiener process defined on the filtered probability space $(\Omega, \_, Q)$ on the interval $[0, T]$; where Q stands for the equivalent martingale measure (also known as risk-neutral measure), under which discounted prices are martingale. The solution to the stochastic differential equation (1) has an analytical solution:

$$S_t = S_o e^{\left(r - (1/2)\sigma^2\right) + \sigma W_t} \tag{2}$$

Where S_0 is a given initial value.

We assume that the rates of return of the reference portfolio are given by

$$s_t = \frac{S_t}{S_{t-1}} - 1 \tag{3}$$

in order that $1 + g_t = e^{(r - (1/2)\sigma^2 + \sigma(W_t - W_{t-1}))}$. Here we will work with their logarithms in order to obtain the continuously compounded rates of return, which are normally distributed with mean $r - (1/2)\sigma^2$ and variance σ^2.

Now that the set of assumptions have been described, we explain the policy framework and its fair valuation. Consider a single policy issued at time zero and matures after T years. We assume that the policyholder makes a single-sum payment at the inception of the contract (Premium). If the policyholder dies within the contract's term or stays alive, whichever come first, the insurance company will pay a specific amount of money (C_t). This benefit will increase each year according to the minimum guarantee and participation rate credited to the policyholder (bonus), as shown as follows:

$$C_t = C_{t-1}(1 + \theta_t) \tag{4}$$

where θ denotes the bonus credited at time t. If we denote C_o as the initial benefit insured in case that the policyholder dies during the first year of contract then we can

represent equation (9) as follows.

$$C_t = C_0 \prod_{j=1}^{t} (1 + \theta_j) \qquad (5)$$

The bonus rate θ is defined by as follows:

$$\theta_t = \max\left\{\frac{\alpha\, r_t - g}{1 + g}, 0\right\} \qquad (6)$$

where g denotes the minimum rate of return guaranteed to the policy holder, r_t the annual return of the reference portfolio and α the participation rate. This last parameter is assumed to be constant in time and takes values within $[0,1]$.

In order to calculate the value of C_t represented by equation (4) the martingale approach by Harrison and Kreps (1979) and Harrison and Pliska (1981, 1983) is used. The value of the C_t at time 0 is expressed as:

$$\pi(C_t) = E^Q\left[e^{-rt} C_t\right] \qquad t = 1,2,...T \qquad (7)$$

with $E^Q[\cdot]$ representing the expected value under the risk-neutral measure Q. Then substituting equations (10) and (11) in equation (12) lead us to:

$$\pi(C_t) = C_o \prod_{j=1}^{t}\left(e^{-r} + \frac{\alpha}{1+g} E^Q\left[e^{-r} \max\{1 + r_j) - (1 + g/\alpha), 0\}\right]\right) \qquad (8)$$

since $1 + r_j$ are for all j are distributed in the same way in that the stock price in the model proposed by Black and Scholes(1973). Then, the expectation value in the equation (13) represents the value of an European call option at time 0 on a stock that pays no dividend, and has an initial price equal to 1, strike price equal to $1 + g/\alpha$. If we denote that value as c, then we get:

$$\pi(C_t) = C_o \prod_{j=1}^{t}\left(e^{-r} + \frac{\alpha}{1+g} c\right) \qquad (9)$$

here, c is given by the Black-Scholes valuation formula:

$$c = N(d_1) - (1 + g/\alpha)e^{-r} N(d_2) \qquad (10)$$

with $d_1 = \dfrac{r + (1/2)\sigma^2 - \ln(1 + g/\alpha)}{\sigma}$ and $d_2 = d_1 - \sigma$.

Finally Bacinello obtains the Fair value of the policy (F) by adding together the values at time 0 of C_t for $t = 1,2,...,T$ weighted with the probabilities that the policyholder dies during the t-th year of contract $(q_x(t,t-1))$ or that he is still alive at time T-1 $(p_x(T-1))$:

$$F = C_o \left(\sum_{t=1}^{T-1} q_x(t, t-1) v_*^t + p_x(T-1) v_*^T \right) \qquad (11)$$

where $v_* = e^{-r} + \dfrac{\alpha}{1+g} c$.

3. DETERMINATION OF THE PROFIT

Once that we have described the scheme for the fair pricing of the insurance contract we proceed to discuss the approach used to calculate the profit of the insurance company. Consider that the profit of the company is given by the difference between the company's income and the value of the payments. The income is composed of the total earned premiums (EP) and the return of the investment (II) in the financial market; the value of the payments (L) depends on expected amount of money to be paid out in claims and contractual obligation. We can represent the profit as follows:

$$Profit = EP + II - L \qquad (12)$$

The value of the total earned premiums depends on the premium (P) charged for a single policy and the quantity of contracts sold (n).

$$EP = n\, P \qquad (13)$$

Taking into account that in order to comply with regulation of the insurance business, insurance companies do not invest all the earned premiums in the financial market; we can represent the investment income as:

$$II = \gamma\, EP * R \qquad (14)$$

where γ represents the percentage of the earned premiums to be invested, and R represents the rate of return of the reference portfolio.

Regarding the total payments, their total value depends on the expected amount to be paid out for a single claim and the quantity of contracts sold.

$$L = n\, F \qquad (15)$$

where F represents the expected payments for a single policy.

4. OPTIMIZATION MODEL

We now turn to the formulation of the model. First we describe additional assumptions taken into account in the final model, and then we associate the elements discussed in Section 2 and Section 3 with the purpose of setting up the optimization model. Table 1 show the notation used in the model.

In our analysis the demand is assumed to be a linear decreasing function of price. That is to say, the higher the premium for a policy, the less people will demand that policy:

$$n = aP + b \tag{16}$$

For modeling purpose we suppose that the optimal premium should not fall below a certain amount (P_d), conversely it should not go above a certain bound (P_u). Furthermore, the percentage of the premium to be invested in the financial markets is constant and should not exceed γ_0. In order to calculate the investment income we use the average rate of return R of the reference portfolio during the planning horizon.

Keeping in mind the assumptions described above and considering that the insurance company aims to maximize the profit for the policy under scrutiny we formulate the model as follows:

$$\textbf{\textit{Max}} \; Profit = EP + II - L \tag{17}$$

s.t

$$P_d \leq P \leq P_u \tag{18}$$

$$\gamma \leq \gamma_0 \tag{19}$$

$$n = a \, P + b \tag{20}$$

Subsequently we think about the expected payouts for a single policy (F) as the fair value of the policy explained in section 2, at the same time this value represents the lower bound for the Premium. Also we consider that the policy is addressed to individuals that at the inception of the contract will be x years old. Finally, the model is given by:

$$\textbf{\textit{Max}} \; n_x P_x + \gamma(n_x P_x) * R - n_x C_o \left(\sum_{t=1}^{T-1} q_x(t, t-1)v_*^t + p_x(T-1)v_*^T \right) \tag{21}$$

s.t

$$P_d \leq P_x \leq P_u \tag{22}$$

$$\gamma \leq \gamma_0 \tag{23}$$

$$n_x = a_x P_x + b_x \tag{24}$$

5. NUMERICAL RESULTS

In this section we discuss the results from the numerical results of the model. Firstly we defined the values of r, g, α based on the solutions for the fair value carried on by Bacinello(2001). Since the insurance companies invest most of the premium collected we set γ_0 to 85%. The rest of the parameters are shown in table 2.

We solve the model using the Soft Approach method put forward by Xu(2006) . The results are as follows:

Optimal Premium = $8,980
Fair Value = $7,370
Number of clients = 10,490
Percentage of the premiums invested = 84.89%
Earned Premiums = $9, 419, 4769

Investment Return = $11,995,384
Profit = $28,879,179

6. CONCLUSIONS AND REMARKS

In this paper we have introduced a methodology which we believe represents a more realistic approach for deciding the premium of a life insurance contract. We developed a model that maximizes the insurer profit of life insurance endowment policy paid by a single premium at the inception of the contract. It is important to remark that in our model uses the average rate of return for calculating the investment income; therefore an extension of our model would be incorporating stochastic rates of return of the reference portfolio. In our model we used the methodology proposed by Bacinello(2001) for the fair pricing of the contract; however an interesting topic to be addressed would be incorporate a different approach for the fair pricing and link it with stochastic rates of return, in order to decide not just the premium but also critical parameters such minimum guarantee and participation rate.

Table 1. Notation used for the model

PARAMETERS	
$C_0 \rightarrow$	Initial sum insured.
$x \rightarrow$	Age of the insured at the inception of the contract
$q_x(t, t-1) \rightarrow$	Probability that the insured dies during the t-th year of contract.
$p_x(T-1) \rightarrow$	Probability that the insured is alive at time T-1.
$T \rightarrow$	Maturity time of the contract
$r \rightarrow$	Compounded market rate
$g \rightarrow$	Guarantee rate
$\alpha \rightarrow$	Participation rate
$\sigma \rightarrow$	Volatility of the reference portfolio
$R \rightarrow$	Average rate of return of the reference portfolio during the planning time
$N(\cdot) \rightarrow$	Cumulative distribution of a standard normal variable
$\gamma_0 \rightarrow$	Upper bound for γ
$a, b \rightarrow$	Coefficients of the demand function
DECISION VARIABLES	
$P \rightarrow$	Premium
$\gamma \rightarrow$	Percentage of the premium that is to be invested on a reference portfolio
DEPENDENT VARIABLES	
$n \rightarrow$	Number of clients
$EP \rightarrow$	Earned premiums
$II \rightarrow$	Investment Income
$L \rightarrow$	Total Payments
$F \rightarrow$	Expected payments for a single contract

Table 2. Parameters

$C_0 \rightarrow$	\$10,000
$x \rightarrow$	40 years
$q_x(t, t-1) \rightarrow$	From United States actuarial life tables(as for 2002)
$p_x(T-1) \rightarrow$	From United States actuarial life tables(as for 2002)
$T \rightarrow$	15 years
$r \rightarrow$	3%
$g \rightarrow$	2.5%
$\alpha \rightarrow$	15.14%
$\sigma \rightarrow$	20%
$R \rightarrow$	15%
$N(\cdot) \rightarrow$	From cumulative normal distribution tables
$a \rightarrow$	-4.4
$b \rightarrow$	5,000
$\gamma_0 \rightarrow$	85%
$P_u \rightarrow$	\$9000

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