

# DECISION ANALYSIS OF INFORMATION COMPLETENESS PROBLEM

**Ryohei Matsumura**

Toyo University: 112-8606 5-28-20 Hakusan Bunkyo-ku Tokyo Japan  
matsumura@toyonet.toyo.ac.jp

## ABSTRACT

The information completeness problem involves a decision making process in which an agent is given correct information about the impact of his/her conduct both internally and externally and is encouraged to elevate his/her morals in order to enhance the level of utility of that agent, as well as of organizations and society as a whole. Potential utility is a new concept for measuring this level of utility, which stated precisely, is the utility perceived by agents possessing sufficient information regarding the problem. In this study, I aim to apply this model to drunk-driving problem. The major findings obtained from the model analysis are as follows.

1. It is effective to expend a high cost towards preparing complete information for the agents who are under the impression that drunk-driving does not have a significant negative impact on society.
2. It is not effective to expend a high cost towards preparing complete information for agents who are aware that drunk-driving has a significant negative impact on society, but who perceive a significant loss of utility in stopping it.
3. It is effective to expend a high cost towards preparing complete information for agents who are highly interested in the safety benefits for society due to reducing drunk-driving and who attach importance to ethical behavior.
4. If the concavity of positive utility function is larger than that of the agent's cost function and the concavity of degree of information completeness is larger than that of the principal's cost function, then proposition 1-3 may hold. (conjecture)

Keywords: information completeness problem, potential utility, drunk-driving, agency model, autonomous agency model

## INTRODUCTION

The *information completeness* problem (the cost of preparing complete information for a decision-making process) can be handled using the Potential Utility model. The objective of the research described in this paper was to apply a potential utility model to a problem which has been receiving particular attention recently, that of traffic accidents due to drunk-driving.

The cost of preparing complete information is the cost required of an organization (the nation state, civic groups, *etc.*) for the creation of advertisements, pamphlets and so on in order to supply correct information regarding the influence of individuals' behavior on

## Decision analysis of information completeness problem

society. Such activities facilitate the voluntary moral advancement of individuals, and increase the level of satisfaction not just for the individual, but for organizations and society as a whole. Potential utility is a new concept for measuring this level of satisfaction (utility), which stated precisely, is the utility perceived by agents possessing sufficient information regarding the problem.

The author and colleague have proposed the *potential utility* model in (Matsumura,2007 Matsumura and Kobayashi,2005) and have applied it to some problems such as environment (the reduction of carbon dioxide emissions) and information ethics (illegal software duplication). In the present paper I analyzed a model arranged functions' form pervious model, thus proving the robustness of our research propositions.

The structure of this paper is as follows. Simple examples are given to explain the structure of the problem so that readers may have a more concrete image in the next section. In the third section it is explained where the potential utility model stands among existing research. The basic concepts of the model are stated in the fourth sections. Conclusions are stated in final section.

The major findings obtained from the model analysis are as follows.

1. It is effective to expend a high cost towards preparing complete information for civilians who are under the impression that drunk-driving does not have a significant negative impact on society.
2. It is not effective to expend a high cost towards preparing complete information for civilians who are aware that drunk-driving has a significant negative impact on society, but who perceive a significant loss of utility in stopping it.
3. It is effective to expend a high cost towards preparing complete information for civilians who are highly interested in the safety benefits for society due to reducing drunk-driving and who attach importance to ethical behavior.
4. If the concativity of positive utility function is larger than that of the agent's cost function and the concativity of degree of information completeness is larger than that of the principal's cost function, then proposition 1-3 may hold. (conjecture)

## EXAMPLES

### Education of children

Normally, parents would like their children to make better decisions. Children, however, often make decisions that they later regret in the long run because they only have a limited range of information and knowledge. Having learnt a lot about life, it is possible for parents to know much more about the nature of the problem that their children are facing than the children do. Parents in such cases can take time (i.e. cost) to give advice to the children or try to persuade them. Then, the children, if convinced by the advice, will behave as instructed, and will do otherwise if they are not convinced. However, parents normally face a lot of decision-making problems besides their children's education, and therefore, they are not willing to spend a limitless amount of time paying

## **Decision analysis of information completeness problem**

attention to their children or for persuading them. Potential utility model is intended to analyze how much time parents should spend for giving advice or education in such situations.

### **Information ethics**

Regarding the problem of illegal software copying, for example, many of those who are making such copies seem to consider that their actions do not have much impact on society and continue to do so obliviously. In addition, it is expected that a considerable number of people would stop or reduce the extent of such conduct if they really understood the fact that their "illegal copying will cause significant problems on copyright and incentive, and in future, this will have an impact on themselves, as well as on society". This problem can be also handled by our model.

## **POSITION OF THE POTENTIAL UTILITY MODEL**

This section identifies the position of the potential utility model among conventional academic disciplines. The model can be considered as a type of developed principal-agent model (hereafter, the agency model).

A conventional standard agency model addresses the way in which pecuniary incentive should be given in conditions where one economic subject (a Principal) asks another economic subject (an Agent) to behave to meet her purpose (Spremann,1987). Usually, the Principal is assumed to be unable to judge the Agent's behavior (level of effort) accurately from the outcome of the Agent. In such conditions, the Agent may not necessarily assume to behave that the Principal desires, but may be lazy. To prevent such "*moral hazard*", it is effective to link money incentives with outcomes strongly, that is, to introduce so-called performance-based wage. However, this method may burden the Agent with great risk: if it is presumed that the opportunity cost (the reservation utility) which is decided by labor market conditions must be given to the Agent to hold the contract, the Principal must disburse the corresponding risk premium. It is one of the main purposes of a conventional standard agency model to analyze how to provide pecuniary incentive to maximizes the utility of the Principal in such conditions.

However, only money does not motivate members of an organization. Many researchers and practitioners claim that the fun or challenge of a job are also important motivational factors. I and a colleague proposes a model that also analyzes the decision making issue about how much effort for promoting such intrinsic motivation should be made (Matsumura and Kobayashi,2004).

The structure of the model to be proposed is the following: the Principal informs the Agent how to give pecuniary incentive according to the outcome, and the motivation cost. Then, the Agent determines his behavior to maximize the value of his utility function under this incentive system and motivation cost. In view of this, the Principal determines the pecuniary incentive system and the motivation cost so that the value of her utility

## Decision analysis of information completeness problem

function will be maximized

In potential utility model, a principal controls the agent *by providing information*, while the agent simply makes its own decisions *autonomously* according to the information given by the principal. Unlike typical agency models, the principal's objective function is almost the same as the agent's own utility function. Only the provision of information is conducted to encourage the agent to make autonomous and subjective decisions. Thus we can say that the potential utility model is the *autonomous agency model*.

### MODEL

#### Basic structure

It is assumed that a set of alternatives is clear to the agent facing a decision making moment to select the most desirable one (Clemen,1996 Mas-Colell,1995 Rubinstein,1997). Here, we do not treat the state of incomplete information, where the agent is not certain about the alternatives (some models have been proposed that take into consideration the state of incomplete information at this level, including hyper game and drama theory (Benett,1980 Kijima,1996), but such theories do not take into consideration at all the state of incomplete information in the terms considered in the potential utility model). Regardless of whether or not he/she knows that his/her information is incomplete, the agent uses the available information to estimate the results that the alternatives will bring about, and gives a value to each of the probable results for assessment. The very correspondence between the alternatives and the evaluations is the agent's utility function. Then, the agent selects the alternative that maximizes the value of the utility function. However, the information available to the agent about the cause-effect relationship between the possible actions and results is not always correct, in general, and he/she may eventually make a selection that will only result in low utility when he/she "*gets wise after the event*", even though the agent tries to maximize the utility.

This "*gets wise after the event*" is one of the key points of the potential utility model. It means the utility that the agent perceives after making a decision, obtaining a result, and then owning sufficient information about the relevant problem. We call this the potential utility. Referring to the example of parents and children in the second section, the sense of value when they are seeking short-run utility, without thinking about the future, is the decision-making utility, whereas the utility when advised by parents is the potential utility. In the example of illegal copying, the utility when obviously making illegal copies without knowing the impact on society is the decision-making utility, whereas the utility when the agent understands the impact after being provided with complete information about the relevant problem is the potential utility.

## Decision analysis of information completeness problem

### Application model

The potential utility model is applicable to various arguments; this section discusses the problem of the degree of information completeness. The problem of information completeness involves a decision making problem in which a person is given correct information about the impact on the external society as a result of the person's conduct in the long term in order to encourage the person to independently elevate his morals to ensure, in turn, enhanced satisfaction of the person, as well as the whole organization or society. If the provision of complete information requires no cost, it is easy to make a decision. It is preferable to maximize the level of completeness to make the person fully aware of the true utility. In reality, however, provision of complete information is costly, and so it is not suitable to promote enlightenment until the person becomes fully aware of the potential utility. Therefore, it should be clear that it is important to analyze the decision making problem of "how much cost should we bear for providing complete information?". The following will analyze the problem of making a decision in terms of how much should be paid for public relations activities and so on against the deleterious effect of drunk-driving.

Next, a model specialized for the problem of drunk-driving is exhibited. The decision-making process is modeled mathematically as follows.

1. The principal (with complete information) sponsored by the government or non-profit organizations provide correct information to civilians (agents).
2. Civilians' utility functions change in response to the cost expended, drawing closer to their potential utility.
3. Civilians make choices during the decision-making process that maximize the utility function.
4. The principal perform decision-making regarding the cost of preparing complete information, in order to maximize the difference between social welfare (the sum of civilians' potential utility values) and the expended cost.

This may be expressed as a specific decision-making model as follows.

$$\begin{aligned} \max_c & U^*(e) - c/a^2 \\ \text{s.t.} & e \in \arg \max_e U(e) \end{aligned}$$

$U^*(e) - C(c)$  Å Principal's utility  
 $c$  Å Principal's decision-making variable  
 $U(e)$  Å Expected utility when agents perform decision-making  
 $U^*(e)$  Å Potential utility  
Å@Å@Å@ : Agents' level of effort towards ethical behavior

## Decision analysis of information completeness problem

ÄÄÄÄÖ@

The notations appearing in each function are explained in order below.

$e$  ÄÄÄÄE The extent to which agents (civilians) make an effort to stop drunk-driving (effort level)

$e$  is a variable expressing the effort level.  $e$  takes a high value when for example, agents are prepared to take a train or bus to places where there is a possibility of consuming alcohol despite some degree of inconvenience, or alternatively, when they have consumed alcohol they are prepared to take a taxi or employ a stand-in driver despite the corresponding cost. Naturally, the risk of traffic accidents is decreased when this variable takes a large value.

$k$  ÄÄÄÄE The beneficial effect on social welfare corresponding to the level of effort made towards stopping drunk-driving

$k$  takes a large value when individuals make efforts to stop drunk-driving, traffic accidents are significantly reduced and the resultant improvement to social welfare is also large. Conversely,  $k$  takes a low value when the rate of traffic accidents does not change a great deal despite individuals making a lot of effort. In practice, it is extremely difficult to measure this value, but the model is idealized so the result is considered measurable, and is modeled using the real number  $k$ .

It is desirable to consider  $k$  as an exogenous objectively determined variable (parameter), such that for example, by paying only an extra 1 Yen and using taxis or stand-in drivers the rate of traffic accidents may be reduced by  $k$  percent, or as an alternative example, by refraining from 1ml of alcohol the field of view may be increased by  $k$  percent. Agents with incomplete information do not know the correct value of  $k$  so they hold a belief about its subjective value. The value of  $k$  reflects agents' initial information, and  $k^*$  represents the true value.

$ak \log e$  ÄÄÄÄE The positive utility obtained by agents

When the level of effort is  $e$ , traffic accidents are in practice reduced, and social welfare is improved by  $k^* \log e$ . On the other hand, the improvement in social welfare with respect to traffic that agents themselves presuppose is denoted  $k \log e$ . The so-called sense of satisfaction, or positive utility  $ak \log e$  that agents perceive is taken as drawing from the improvement to social welfare based on this presupposition.

$a$  ÄÄÄÄE Parameter expressing how easy it is to perceive the positive utility granted by a reduction in traffic accidents

If  $a$  is large, then a great sense of satisfaction is obtained from a reduction in traffic accidents, and the agents can be assumed to attach a high value to so-called ethical and

## Decision analysis of information completeness problem

procedural utility. If  $a$  is instead small, it can be assumed that they are comparatively more concerned by other forms of utility (related to different day-to-day problems for example) rather than ethical utility.

$d$   $\hat{=}$  Disutility perceived by agents

The disutility perceived by agents making an effort level  $e$  towards not drunk-driving is taken as  $d e$ .

In previous research we adopted other function forms. The agent's positive utility is linear and agent's cost is convex. Both the present and previous research has found the concavity of positive utility function is larger than that of the agent's cost function.

$d$   $\hat{=}$  Parameter expressing how easy it is to perceive the disutility caused by behavior that avoids drunk-driving

$d$  is large when maintaining conduct that avoids drunk-driving is easily perceived as tiresome. If on the other hand, disutility is not perceived even when  $e$  takes a large value then  $d$  takes a small value.

Some relationships among the values of  $a$  and  $d$  would be exist, but no particular functional relationship is enforced.

$U(e) = U_k(e) = a k \log e - d e$   $\hat{=}$  The utility of agents during the decision-making process

This varies according to the information held by agents,  $k$ . For example, if mistaken beliefs are held regarding the variations in social welfare caused by drunk-driving, it implies that correspondingly distorted utility functions are envisaged. For example, if refraining from 1ml of alcohol can actually reduce traffic accidents by 1 percent, then if the *supposed* reduction is only 0.1 percent, the agent's positive utility during the decision-making process obtained by refraining from drunk-driving is reduced by a factor of 10.

$U^*(e) = U_{k^*}(e) = a k^* \log e - d e$   $\hat{=}$  The utility perceived by agents with completely prepared information

This is the so-called true utility that ought to be perceived by agents who make an effort level of  $e$ , and for whom time has passed or instruction has been received, and who possess complete information (after-the-fact). This is why we have applied the name potential utility. Even when decision-making is performed based on the utility function possessed at the time of decision-making, there are many cases for which after some time there comes a point when people have a realization that "*ah, it was like that...*". The utility at that point is the potential utility.

$a \log c / d c_{\max}$   $\hat{=}$  The degree to which complete information is prepared

$c$  has a high value when many agents have sufficient knowledge of the problem in question. Indeed, one can say that in such cases there are many people who understand the precise influence of drunk-driving on society as a whole. It is assumed that agents'

## Decision analysis of information completeness problem

beliefs can be converted to the true value  $k^*$  with probability  $a \log c / c_{\max}$ . I assume that making the value of  $c$  large requires a large cost.

### The cost of instruction

It is assumed that the cost required to maintain the probability of successful instruction at a level of  $a \log c / dc_{\max}$  is  $c$ . The marginal cost is thus assumed to be increasing. In this case the analytical convenience is considered and such functions are adopted. The next issue will be the relaxation of functional constraints. The cost includes advertisements, pamphlets and suchlike.

In previous research we adopted other function forms. The degree of information completeness is linear and the principal's cost is convex. Both the present and previous research has found the concavity of degree of information completeness is larger than that of the principal's cost function.

Decision-making problem was solved according to the settings above.

Firstly, the level of individuals' expected utilities before and after awareness programs are compared. The following procedure is used to obtain the expected utility before instruction.

Solving the following maximization problem at the agents' decision-making,

$$\max_e U_k(e) = ak \log e - de,$$

yields the solution

$$e_{kopt} = ak / d.$$

Under this solution, the agents' true utility (potential utility) is

$$U_{k^*}(e_{kopt}) = ak^* \log(ak / d) - ak$$

On the other hand, after preparing complete information, the information that agents possess is converted into the true value  $k^*$ , with probability  $a \log c / dc_{\max}$ . Solving the following decision-making problem for instructed agents,

$$\max_e U_{k^*}(e) = ak^* \log e - de,$$

yields the solution

$$e_{k^*opt} = ak^* / d.$$

At this point the utility actually obtained by agents is  $U_{k^*}(e_{k^*opt}) = ak^* \log(ak^* / d) - ak^*$ .

It can be seen from the above that when the cost of preparing complete information is  $c$ , the expected value of the improvement in social welfare that motivates preparing complete information is  $a \log c (ak^* \log(k^* / k) - a(k^* - k) / dc_{\max})$ .

The utility of the principal is this expected utility increment, with the cost of preparing complete information subtracted, so the decision-making problem for the principal may be resolved by solving the following optimization problem,

## Decision analysis of information completeness problem

$$\max_c (a \log c (ak^* \log(k^*/k) - a(k^* - k)/dc_{\max}) - c),$$

and obtaining

$$c_{opt} = a(ak^* \log(k^*/k) - a(k^* - k)/dc_{\max}).$$

This is the solution to decision-making problem Ö@

The relationship between  $c_{opt}$  and the parameters  $k - k^*, d, a$  is now considered. It can be seen that the following relationship holds regardless of the values of the parameters.

$$\frac{\partial c_{opt}}{\partial(k - k^*)} > 0, \quad \frac{\partial c_{opt}}{\partial d} < 0, \quad \frac{\partial c_{opt}}{\partial a} > 0$$

## CONCLUSION

Interpreting the implications of this comparative static analysis reveals the following points.

1. Regarding the degree to which drunk-driving influences traffic accidents and social welfare, it makes sense to facilitate a large cost for preparing complete information when the values held by individuals as initial beliefs, and the true values are widely different.
2. When the ease of perceiving disutility due to behavior that avoids drunk-driving is low, it makes sense to facilitate a large cost for preparing complete information.
3. When it is easy to perceive the precise utility due to a reduction in traffic accidents, it makes sense to provide a large cost for preparing complete information.

From the results of both the present and previous research, I have formulated an important conjecture. That is, “if the concavity of positive utility function is larger than that of the agent’s cost function and the concavity of degree of information completeness is larger than that of the principal’s cost function, then proposition 1-3 may hold”.

Of course, this conjecture requires mathematical proof. However, I strongly believe it is correct, and the proof will be the main theme of future research.

Various constraints were applied to the functional forms used in this research in order to facilitate an analytical breakdown. The next problem is analysis with functional forms that have relatively few constraints. Before dealing with this problem, a number of numerical examples are presented. The intention is to reveal a specific image reflecting the implications of comparative static analysis.

## REFERENCES

Benett, P. (1980) “Hypergames: developing a model of conflict”, *Futures*, vol. 12, no. 6.

## Decision analysis of information completeness problem

- Clemen, R. (1996) “*Making hard decision: introduction to decision analysis*”, Duxbury Press.
- Kijima, K. (1996) “Intelligent poly-agent model and its application”, *Information and Systems Engineering*, vol 2.
- Mas-Colell, A., Whinston, M. D., Green, J. R. (1995) *Microeconomic theory*, Oxford university Press.
- Matsumura,R (2007) “A new approach to mathematical ethics – the basis of potential utility theory- ”, *Gendai syakai kenkyu*, pp. 33-40, (Japanese).
- Matsumura,R (2007) “The application of potential utility model for drunken driving problem”, *Toyo university kiyu* 2007, pp. 89-102, (Japanese).
- Matsumura,R., Kijima,K., Nakano,B. and Inohara,T. (1999) “Analysis of the Incentive Problems Considering Intrinsic Motivation and Multiple Objectives of the Agent”, *Proceedings of 1999 IEEE International Conference on Systems, Man, and Cybernetics*, V-1031-1033.
- Matsumura,R. and Kobayashi, N. (2004) “An analysis of agency problem considering intrinsic motivation”, *Communications of the Operations Research Society of Japan*, Vol49, No12, pp.751-755 (Japanese).
- Matsumura, R. and Norimasa, K. (2005) “The Theoretical Foundation of Potential Utility”, *The proceedings of the 1<sup>st</sup> World Congress of IFSR 2005*(CD-ROM).
- Rubinstein, A. (1997) *Modeling bounded rationality*, MIT Press.
- Spremann,K. (1987) ‘Agent and Principal’, in “*Agency Theory, Information, and Incentives*”, Springer-Verlag.