Mexican Voter Network as a Dynamic Complex System

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Abstract

One of the most important properties in the systems is complexity. When a great level of complexity exists in a system, it is considered as a complex system. The complex systems can be soft systems and hard systems.

In hard systems when their elements are interrelated in a non-linear way, they are considered as complex systems, that is to say, complex systems are those that contain a great number of elements interacting in a non-linear way. To try to understand the behavior of this type of systems diverse mathematical tools have been developed. A new scientific discipline with great impact in the analysis of the complex systems has been developed in recent years; we refer to the fractal analysis.

Voting data from Mexican federal deputy elections are analyzed and considered as a response function of a social system with underlying dynamics leading to complex behavior. It was found that voting distributions among candidates, as well as political parties behave as a fat-tail Levy stable distribution, associated with fractal structure of electoral network. Specifically, it is shown that the distribution of voter preferences follows the shifted Pareto distribution with scaling exponent α which shows only a few variations from state to state and it is essentially the same for all federal elections from 1991 to 2003. Furthermore, it is shown that Mexican voter network can be modeled by hierarchical pseudo-fractal network characterized by two different fractal dimensions. The identified hierarchical architecture of voter network offers a new perspective on the analysis, modeling and forecasting of elections.

Keywords: complexity, complex systems, voting network, fractal analysis.

Introduction

Complex behavior can appear in any system composed by a great number of elements interacting in a non-linear way. For example, atoms in a solid, cells in a life-organism, traders in a financial market, members of a political party, religious sect or professional association, or voters in a country or state.

Complex behavior of several systems is the consequence of different processes operating from a wide range of scales (temporal and spatial) which are associated with a great number of freedom degrees (Balankin, 2003). Complexity can be characterized by the uncertainty of the system or by the perfectibility degree of the evolution system. Quantitative analysis of data generated from complex systems is a common topic in statistical physics which finds applications in several areas of natural and social sciences.

Simple models about the cooperative behavior were known by economists and sociologists many years ago. But it is really remarkable that many of those classic models in sociology were simply applied in terms of existent models from statistical mechanics, such as the “Minority Game” (Wiedlich, 2000), the Axelrod’s model of cultural dominions or political coalitions (Schofield, 200), the economic models based on the Nash’s local equilibrium concept, among others (Adams & Merril, 1999).

Efficiency of a system, in terms of information, is closely related to the topology of the network model. In this context, in last decade it has arisen a big interest in the called *complex networks*, which have the property of small-world (Waserman & Faust, 1994), to say, the number of links (n) that must achieved for conecting two sites in the network grows as the logarithm of the sites number (N): $n \propto \ln N$, in contrast with the case of regular networks, where $n \propto N$.

Among complex networks, social networks appear in a natural way, playing an important role. Social networks are composed by a great number of people, who are usually interacting in a local way. Such physical systems connected to external actions and fields, the social networks behavior also depends on external factors. In consequence, mathematical tools developed under the statistical physics context, in order to face collective phenomena, they have being recently applied to several social problems (Waserman & Faust, 1994).

Some of the social networks analized are: (i) relationship networks and close-up social groups, such as actors and political parties; (ii) financial market networks; (iii) electronic mail networks; (iv) scientif colaboration networks; (v) networks of sexual contacts, and many others (Newman, 2003). Recently, the analysis of these networks has shown that the social network structure is not purely random, as the sociology had supposed many years ago, but there are small-world networks, similar to the genetic interactions network, the metabolic networks and the electric networks. In some way, nature has found that this type of structure is optimum in balance between the benefit of one link to a so-far neighbor and the cost that it suposes (Lopez, 2002). So that, social networks often posses typical properties of complex systems studied in physics, such as self-organization, co-operation, and adaptation (Dorogovtsev & Mendes, 2003).

Frequently statistical analysis reveals the emergence of power-law distributions, pointing out self-organized social systems flow towards a critical state without characteristic time or spatial scales (Bak, 1996). Recently it has being reported that several phenomena with scale invariance appear in social systems, for example, in fractal morphologies of cities (Balankin, 2003), in the fractal dynamic of economic phenomena, in the complex dynamic of opinions associated to rumors propagation and electoral preferences (Balankin et al , 2004).

One of the fundamental processes in democratic societies concerns to elections. Elections are democratic processes in which exists the same type of voters interaction and external influences (political publicity, campaigns, etc). From a scientific point of view of complex system, the outcome of a electoral process can be considered as an open system’s response with many elements interacting in a non-linear way and ruled out by an internal complex dynamic (but unknown).

Voters do not possess so much freedom, even in democratic regimens, because individual preferences depend so much on the choice of social networks where the voter is in. This is natural in human being because of different social identities which can cause social tension, so that, microsocial is attractive for reducing social tension by fitting political preferences or ideological approaches. Acoording to that, a cuantitative characterization of the electoral network can be made by studying voter distributions. These distributions can be gotten from different electoral processes and from electoral preference simulations which are indespensable steps towards a better understanding and prediction of the underlying electoral dynamic.

In this work are analized the election outcomes for federal deputies in 1991, 1994, 1997, 2000 and 2003 (IFE, 2005).

Statistical Analysis

Data Analysis

Mexico is composed by 32 federative entities (31 entities and Mexico City) that are autonomous in their way of ruling out, where one of the fundamental processes about democratic societies are elections. Elections are regulated by citizen votes.

This work was made with data proved by Instituto Federal Electoral (IFE), about Mexican people´s votes during 1991, 1994, 1997, 2000 and 2003: 147’409,098 data, which were classified in electoral areas and in entities. Therefore, there were analyzed the statistical distributions of: (1) the number of candidates (N) that received votes (v), (2) the votes between parties, and (3) the votes in every electoral area.

The statistical analysis was supported from the @Risk 4.5 software. This software fits data distributions based on three statistical criteria: Square-Chi, Kolmogorov-Smirnov, and Anderson-Darling. In all cases were determined the distributions that fitted better the data analyzed, according to the three criteria.

By processing information, it was exported from different data bases to Excel software. Three programs developed from the Visual Basic (release 6.0) software, were applied to analyze distributions generated from @Risk 4.5 software: 300 areas during 5 electoral years (1,500 distributions), (2) 32 entities during 5 electoral years (160 distributions), and

(3) 1 country during 5 years (5 distributions). In this way, there were eliminated human mistakes.

Results

It was found that the distributions of votes between neighbors, candidates, and political parties have the same slope and a fat-tail stable behavior related to a fractal structure of the voters network. Moreover, the neighbors, candidates, and parties distributions (figure 1-3) hold the same statistical distribution independently of the electoral year.

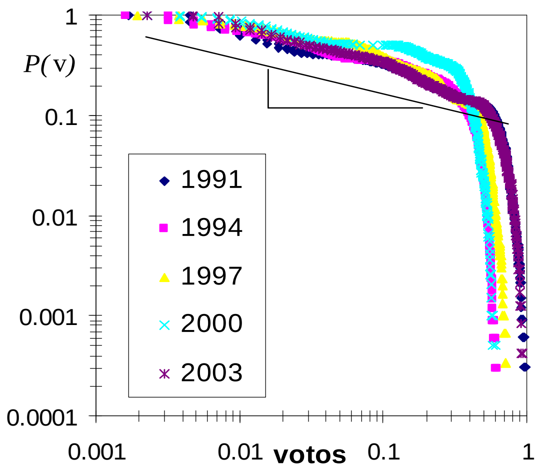

Figure 1. Distribution of votes between neighbors to deputies by relative majority in federal elections 1991-2003.

Specifically, it was found that the best fit for the distributions of votes between candidates (number of participants, *N*, that received a part of votes, *v*, is performed by Pareto's distribution:

$$N_v = \frac{\alpha M^{\alpha}}{v^{\alpha+1}}, \quad v \geq M \tag{1}$$

where $\alpha \geq 0$ is a scaling exponent and M is the mode of distribution. Based on Lévy's criteria, distribution (1) is stable when $0 \leq \alpha < 2$ [16]. When $\alpha > 0$, the cumulative distribution is:

$$F(V \leq v) = 1 - \left(\frac{M}{v}\right)^{\alpha}, \tag{2}$$

so that, the fraction between participants that received a percentage of votes greater than *v* behaves as a power-law:

$$P(V > v) = 1 - F(V \leq v) = \left(\frac{M}{v}\right)^{\alpha}, \tag{3}$$

It implies a linear behavior in log-log axis (figure 1).

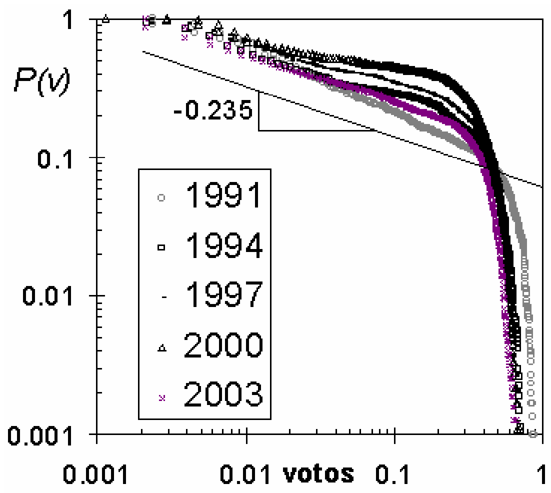

Figure 2. Distribution of votes of candidates to deputies by relative majority in federal elections 1991-2003.

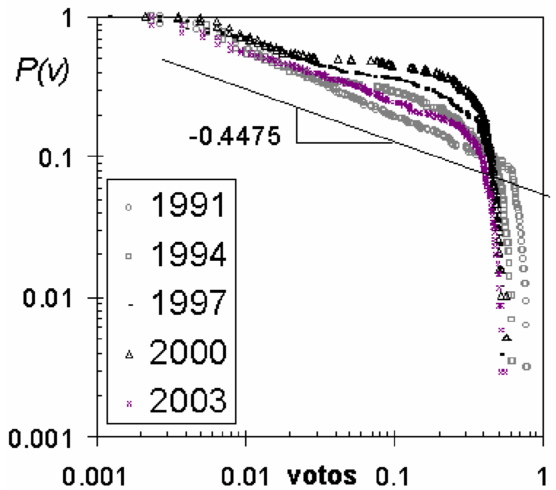

Figure 3. Distribution of votes between political parties in federal elections 1991-2003.

An important aspect of this analysis is that parameters of Pareto's distribution (2), which were obtained from different elections, show only a few variations and the exponent α value is basically the same for every federal elections from 1991 to 2003 (see table 1).

At the same time, it exists a significant difference between the parameters of the distributions of votes by neighbors, candidates, and political parties (see table 1 and figures 1-3).

Moreover, for every election the same statistical analysis was performed in each of the federal entities, finding that the vote distributions, independently of the economical, social, political, and cultural situation of parties and entities, are characterized by the same exponent α_p (see figures 4-5 and table 2).

Table 1. Parameters of the Pareto's distribution of votes by neighbors, candidates, and parties to a federal level.

Year	Neighbors		Candidates		Parties	
	Mode	α_V	Mode	α_C	Mode	α_P
1991	1.85E-03	0.339	1.24E-04	0.195	3.17E-03	0.49
1994	1.65E-03	0.3321	3.20E-04	0.232	3.12 E-03	0.475
1997	2.00E-03	0.3172	4.80E-04	0.24	2.85 E-03	0.408
2000	1.96E-03	0.2915	1.13E-03	0.265	6.16 E-03	0.466
2003	2.53E-03	0.3529	2.99E-04	0.243	2.163 E-03	0.433
Promedio	2.00E-03	3.27E-01	4.71E-04	2.35E-01	3.17E-03	4.54E-01
DS	0.000326246	0.023431026	0.000389601	0.02548529	#¡DIV/0!	0.033306156

Table 2. Median and standard deviation (α) of the exponent of the Pareto's distributions: α_p (distributions of votes by parties in different entities) and α_v (distributions of votes by areas in different entities).

Year	α_P		α_V	
	Median	σ	Median	σ
1991	0.485	0.108	0.340	0.020
1994	0.471	0.093	0.334	0.013
1997	0.400	0.038	0.319	0.010
2000	0.463	0.051	0.292	0.009
2003	0.420	0.068	0.354	0.020
$\pm\sigma$	$\alpha_P = 0.45 \pm 0.04$		$\alpha_D = 0.33 \pm 0.02$	

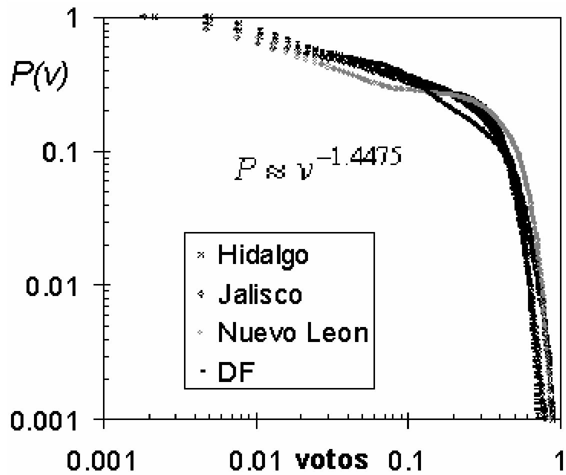

Figure 4. Distribution of votes between political parties in four federal entities during federal elections in 2003.

The results show that the Mexican electoral network is characterized by three exponents, which rule out the distributions of votes between candidates (*α_c=0.24±0.04*), political parties (*α_p=0.45±0.04*), and areas (*α_D=0.33±0.02*). It is important to point out that the values between exponents are practically the same for all areas, all entities and all elections.

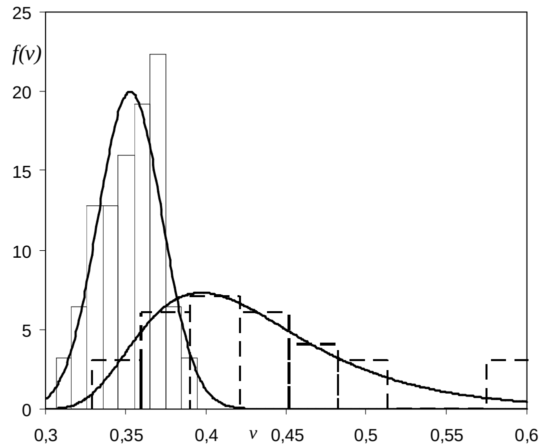

Figure 4. Distribution of the exponent α for the votes between political parties obtained during the period 1991-2003.

Analysis and Discussion of Results

The same trend is observed when citizens vote. This trend is explained by a power-law that models a general social model and this trend is persistent around small government systems. The remarkable similarity in all distributions of votes, even big economic and social differences among all regions in Mexico, is a signal of a common mechanism that exists in the making-decision process.

In statistical analysis about the elections in Brazil (Costa, et al. *a*, 1999), (Costa, et al. *b*, 2003), Indonesia (Situnkir, 2004), Sweden (Alves et al, 2002), and India (Gonzalez et al, 2003), there were proved that the votes between candidates distribution behaves according to a power-law (2), but with different exponents α_p (see table 3). In all those works there were not analyzed the distributions of votes between parties and areas because the electoral systems of those countries are so different respect to Mexico.

Table 3. Exponent $\theta = \alpha_C + 1$ for the vote distributions between candidates in different countries.

Indonesia	India	Mexico	Brazil	Sweden
1.41	1.32	1.24	1.00	0.96

Authors in (Alves et al, 2002) performed numerical simulations of elections, based on the Sznajd model of a pseudofractal network, which reproduces the power-law distribution (1). The differences between the Sznajd simulations about pseudofractal networks about the elections happened in Brazil and India are consequences mainly because of the number of candidates, *N*, taken into account in each one. In Brazil *N* almost represents 0.005% of nodes in the mall, and for the former is 0.01%. The faster the fix point (agreement) is achieved, the greater is the candidates' density.

On the other hand, it is appreciated that many of social networks share two general properties: (1) they do not posses a characteristic scale (free-scale), and (2) they display a high degree of clustering. Both of them are consequence of a hierarchic organization, implying that small groups of nodes are organized into bigger groups in a hierarchic way, only if a free-scale topology is kept. In a free-scale network, the probable degree of diverse nodes follow a power-law distribution is given by [22]:

$$P(k) = ck^{-\gamma} \text{ for } k_0 \le k \le K \qquad (4)$$

where *c* is an appropriate factor of normalization, α is the exponent of the distribution of connections, k_o is the minimum degree of any node, and the cut degree, *K*, depends on the size of the network *N* as $K = k_0 N^{1/(\gamma-1)}$ [22]. A greater value of α, leads to an insignificant connection in the network.

For a fractal network, α is related to the distribution exponent (1) as (Cowan et al,1994):

$$\gamma = 1 + \frac{1}{\alpha} \qquad (5)$$

Besides, if $3 < \gamma \le 4$, so the fractal dimension of a free-scale network, D_F, is related to α as [22_21]:

$$D_F = 2\frac{\gamma-2}{\gamma-3}, \text{ and then, } D_F = 2\frac{1-\alpha}{1-2\alpha} \tag{6}$$

But if $\gamma > 4$, so the fractal dimension of the network has a universal value of $D_F=4$.

In sum, the difference between values of Pareto's exponent for the distribution of votes (in diverse countries) can be attributed to the difference in fractal dimension (see table 4).

Table 4. Exponent of connectivity and fractal dimension in electoral networks in different countries.

	Indonesia	India	Mexico		
			Total	Neighbors	Parties
α	3.44	4.13	5.17	4.03	3.22
D_F	6.56	4	4	4	11

It should be pointed out that the Mexican voter network is characterized by different connectivity exponents in both in a local (neighborhood) and global (entity) scale. It is easy to understand whether it is considered that neighbor voters are less connected than those voters in the global network. It can be appreciated that Mexican voters are less connected than Indian and Indonesian voters (table 3).

Furthermore, it was found that the Mexican electoral network is characterized by two fractal dimensions: the universal ($D_F=4$) and the fractal dimension of votes towards parties included in the network. ($D_F=11$). The biggest fractal dimension of votes points out a strong discipline in political organizations, which leads to deterministic voting of the parties' members ("hard vote" efect).

Conclusions

Among a wide range of complex dynamic systems, two classic problems are essential for the quality democracy: opinion forming and voting processes. Elections are processes where many people often interact. In these convincing dynamic processes, in which there are simultaneously neighbors interacting and external influence (political publicity, campaigns, etc), people can vote directly for candidates or for the party ("hard vote").

The hierarchic architecture of the voter network gives a new perspective in the analysis, modeling, and prediction of elections, because it has three scaling exponents: α_v, α_c, and α_p (neighbors, candidates, and parties). So, the distribution of votes between participants is independent of the electoral year, as well as the economic, social and cultural level. Nevertheless, under a social network, voters in a democratic regimen are not completely free due to individual preferences that depend so much on the context.

The vote distributions in Mexico are ruled out by fat-tail distributions, specifically, by a Pareto's distribution associated to voter network structure that displays fractal properties with major density in the voter network in a party level ($D_F=11$) than in a candidate and neighbor levels ($D_F=4$).

In this stage of researching it is emphasized the fact that in the social and political systems modeling there is not an absolute truth, but there are some basic trends in very complex situations.

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References

Adams, J. and Merrill, S. (1999). *Modeling Party Strategies and Policy Representation in Multiparty Elections: Why Are Strategies So Extreme?* American Journal of Political Science, V. 43, : 765-791.

Alves, S. et al (2002).*Electoral Surveys on the Voting Processes: a cellular automata model*, arXiv:cond-mat/0204139.

Bak, P. (1996).*How Nature Works: The Science of Self-Organised Criticality*. Copernicus Press, New York.

Balankin, A. (2003). *Fractal Behavior of Complex Systems*, Científica, V. 7, No. 3:109-128.

Balankin, A. (2004).et al *Crossover from antipersistent to persistent behaviour in time series possessing the generalized dynamic scaling law*, Review, Ser. E , V. 69, No. 3.

Costa Filho, R et al (2003).*Brazilian elections: voting for a scaling democracy*, Physica A, V. 322, :698 – 700 .

Costa Filho, R. et al (1999). *Scaling behavior in a proportional voting process*, Physical Review, Ser. E, V. 60, No. 1:1067-1068 .

Cowan, G. et al (1994). *Complexity, Metaphors, Models and Reality*,. Addison-Wesley, Santa Fe.

Dorogovtsev, S. N. and Mendes, J. F. F. (2003). *Evolution of networks: From biological nets to the Internet and WWW* . Oxford University Press, Oxford.

Galam, S. (2000). *Application of Statistical Physics to Plitics*, arXiv:cond-mat/0004306.

Gonzalez, M. et al (2003).*Opinion Formation on a Deterministic Pseudo-fractal Network*, arXiv:cond-mat/0307537.

IFE (2005) http://www.ife.org.mx

López, J. M. (2002).*Dinámica de la opinión pública en redes sociales de influencia*, Revista Española de Física, V. 16, No. 5, : 59-62

Newman, M. E. J. (2003). *Handbook of Graphs and Networks: from the Genome to the Internet*, edited by S. Bornholdt and H. G. Schuster Wiley-VCH, Berlin

Schofield, N. and Parks, R. (2000).*Nash Equilibrium in a Spatial Model of Coalition Bargaining*, Mathematical Social Sciences, V. 39,:133-174.

Shlesinger, M et al (1995).*Lévy Flights and Related Topics in Physics*, Springer-Verlag, Berlin.

Situngkir, H. (2004).*The Power-law Signature in Indonesian Legislative Election*. Preprint: arxiv: nlin.AO/0405002.

Stauffer, D. (2003).*Sosiophysics Simulations*, Computing In Science & Engineering, No. 3 : 71-75.

Wasserman, S. and Faust, K. (1994). *Social Network Analysis*. Cambridge University Press, Cambridge, U.K.

Weidlich, W. (2000). *Sociodynamics: A Systematic Approach to Mathematical Modeling in the Social Sciences*. Harwood Academic Publishers,

Weidlich, W. (2002). *Modeling Complexity in Economic and Social Systems*, ed. F. Schweitzer World Scientific Singapore.