

# **A SYSTEMS-THEORETICAL REPRESENTATION OF TECHNOLOGIES AND THEIR CONNECTIONS**

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## **ABSTRACT**

This paper proposes a systems-theoretical representation of technologies. A technology is represented as an efficient input-output (I/O) system in the sense of mathematical systems theory, where the I/O system transforms the inputs provided for it through the input channels of it into the outputs, which are outputted from it through the output channels of it. This paper also provides a definition of connections of I/O systems as a way to construct a bigger I/O system from smaller I/O systems. Of course it is not always true that a connection of I/O systems is a technology. It can be verified, however, that a connection of technologies is always a technology. In this paper a mathematical verification of this fact is provided.

Keywords: I/O systems; technologies; connections of technologies

## **INTRODUCTION**

The aim of the research in which this paper is involved is to develop a mathematical method for evaluating technologies, in particular, patents (Razgaitis (1999), Smith and Parr (1994)). As a first step of this research, a systems-theoretical (Mesarovic et al (1974), Mesarovic and Takahara (1975), Mesarovic and Takahara (1989)) representation of technologies will be proposed in this paper. This can be employed for further development of the research: for example, such concepts as ‘networks of technologies,’ ‘fungibility of technologies,’ ‘commercializability of technologies’ and ‘connectability of technologies’ can be mathematically expressed.

In this paper, a technology is represented as an input-output (I/O) system in the sense of mathematical systems theory, where the I/O system transforms the inputs provided for it through the input channels of it into the outputs, which are outputted from it through the output channels of it. It is required for an I/O system to be a technology that the I/O system satisfies that for each input there exists an output such that the input is transformed into the output by the I/O system, and for each output there exists an input such that the I/O system transforms the input into the output.

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In this paper, moreover, a definition of the concept of ‘connectability of I/O systems’ will be provided, and a proposition which claims that a connection of technologies satisfies the conditions to be a technology will be verified.

The structure of this paper is as follows: in the next section, Section 2, the mathematical framework for treating I/O systems and their connections will be provided. The definition of technologies will be given in Section 3, and subsequently, the proposition mentioned above will be verified in Section 4. The last section, Section 5, is devoted to the conclusive remarks.

### MODELS: I/O SYSTEMS AND THEIR CONNECTIONS

This section gives the mathematical framework employed in this paper. This framework is constructed based on the mathematical systems theory (Mesarovic et al (1974), Mesarovic and Takahara (1975), Mesarovic and Takahara (1989)).

Let  $C$  be the set of all I/O channels. For  $c \in C$ ,  $X_c$  is the input set of channel  $c$  and  $Y_c$  is the output set of channel  $c$ .

**Definition 1 (The field of I/O systems)** The field  $F$  of I/O systems is a tuple  $(C, (X_c)_{c \in C}, (Y_c)_{c \in C})$ .

That is, the field  $F$  of I/O systems consists of the set  $C$  of all I/O channels and the input sets  $X_c$  and output sets  $Y_c$  for all  $c \in C$ . Within the field  $F$  of I/O systems, I/O systems are defined as follows:

**Definition 2 (I/O systems)** An I/O system  $t$  is a tuple  $(S^t, I^t, O^t, (X_c^t)_{c \in I^t}, (Y_c^t)_{c \in O^t})$  such that  $S^t \subset X^t \times Y^t$ , where  $X^t = \prod_{c \in I^t} X_c^t$  and  $Y^t = \prod_{c \in O^t} Y_c^t$ .

For an I/O system  $t$ ,  $I^t \subset C$  is the set of all input channels of  $t$  and  $O^t \subset C$  is the set of all output channels of  $t$ . Moreover, for an I/O system  $t$ , an input channel  $c \in I^t$  of  $t$  and an output channel  $c \in O^t$  of  $t$ ,  $X_c^t (\subset X_c)$  is the input set of channel  $c$  of  $t$  and  $Y_c^t (\subset Y_c)$  is the output set of channel  $c$  of  $t$ .

**Definition 3 (Connectability of I/O systems)** An I/O system  $t = (S^t, I^t, O^t, (X_c^t)_{c \in I^t}, (Y_c^t)_{c \in O^t})$  is said to connect with an I/O system  $u = (S^u, I^u, O^u, (X_c^u)_{c \in I^u}, (Y_c^u)_{c \in O^u})$  on  $D$ , where  $\phi \neq D \subset C$ , if  $\phi \neq O^t \cap I^u = D$ . If an I/O system  $t$  connects with an I/O system  $u$  on  $D$ , then an I/O system  $v$  is the connection of  $t$  and  $u$  on  $D$ , denoted by  $tu|_D$ , if and only if  $v = (S^v, I^v, O^v, (X_c^v)_{c \in I^v}, (Y_c^v)_{c \in O^v})$  (see Figure I), where

1.  $I^v = I^t \cup (I^u \setminus D)$ ,
2.  $O^v = (O^t \setminus D) \cup O^u$ , and

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3.  $(x^v, y^v) \in S^v$  if and only if there exists  $y^t \in Y^t$  such that  $(x^v|_{I^t}, y^t) \in S^t$ ,  $((y^t|_D, x^v|_{I^u \setminus D}), y^v|_{O^u}) \in S^u$ , and  $y^v|_{O^t \setminus D} = y^t|_{O^t \setminus D}$ , where  $x^v = (x^v|_{I^t}, x^v|_{I^u \setminus D})$  and  $y^v = (y^v|_{O^t \setminus D}, y^v|_{O^u})$ .

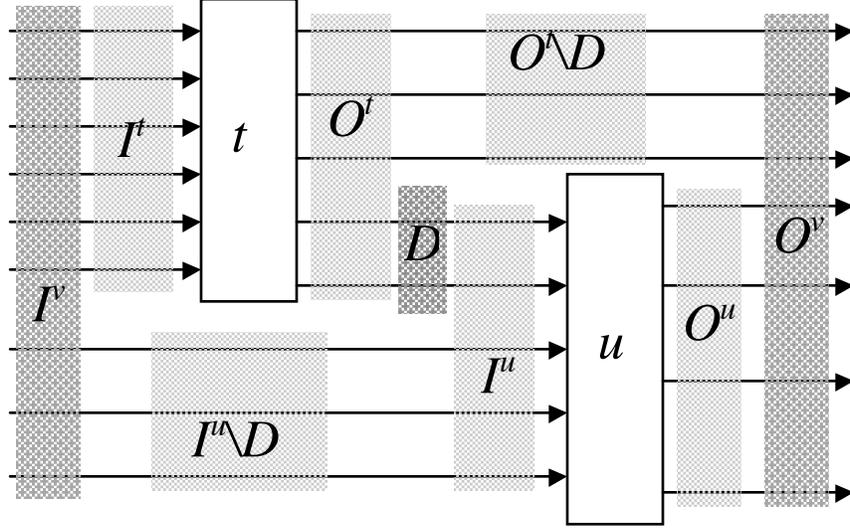


Figure I. The connection  $tu|_D$  of  $t$  and  $u$  on  $D$ .

### TECHNOLOGIES

In this paper, it is required for an I/O system to be a technology that the I/O system is efficient in the sense that for each input there exists at least one output such that the input is transformed into the output by the I/O system, and for each output there exists at least one input such that the I/O system transforms the input into the output. The next is a precise definition of technologies.

**Definition 4 (Technologies)** A technology  $t$  is an I/O system  $(S^t, I^t, O^t, (X_c^t)_{c \in I^t}, (Y_c^t)_{c \in O^t})$  such that;

1. for all  $x^t \in X^t$ , there exists  $y^t \in Y^t$  such that  $(x^t, y^t) \in S^t$ , and
2. for all  $y^t \in Y^t$ , there exists  $x^t \in X^t$  such that  $(x^t, y^t) \in S^t$ .

Another type of technologies, which satisfy a condition that is weaker than the one that is required for an I/O system to be a technology, can be defined as follows:

**Definition 5 (Weak technologies)** A weak technology  $t$  is an I/O system  $(S^t, I^t, O^t, (X_c^t)_{c \in I^t}, (Y_c^t)_{c \in O^t})$  such that;

1. for each  $c \in I^t$  and each  $x_c^t \in X_c^t$ , there exists  $x_c^{t'} \in X_c^t$  such that  $x_c^{t'} = x_c^t$  and there exists  $y^t \in Y^t$  such that  $(x^t, y^t) \in S^t$ , and

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2. for each  $c \in O^t$  and each  $y'_c \in Y'_c$ , there exists  $y'' \in Y^t$  such that  $y''_c = y'_c$  and there exists  $x' \in X^t$  such that  $(x', y') \in S^t$ .

### PROPOSITIONS

The first proposition shows that a technology always satisfies the conditions to be a weak technology.

**Proposition 1** If an I/O system  $t = (S^t, I^t, O^t, (X'_c)_{c \in I^t}, (Y'_c)_{c \in O^t})$  is a technology, then  $t$  is also a weak technology.

**Proof:** For  $c \in I^t$  and  $x'_c \in X'_c$ , one can have  $x'' \in X^t$  such that  $x''_c = x'_c$ , taking  $x''_c$  for each  $c' \in I^t$  such that  $c' = c$  arbitrary. Then, there exists  $y' \in Y^t$  such that  $(x'', y') \in S^t$ , because  $t$  is a technology.

For  $c \in O^t$  and  $y'_c \in Y'_c$ , one can have  $y'' \in Y^t$  such that  $y''_c = y'_c$ , taking  $y''_c$  for each  $c' \in O^t$  such that  $c' = c$  arbitrary. Then, there exists  $x' \in X^t$  such that  $(x', y'') \in S^t$ , because  $t$  is a technology.

The next is the main proposition of this paper, which verifies that a connection of technologies is also a technology.

**Proposition 2 (Connection of technologies is a technology)** If an I/O system  $v$  is the connection  $tu|_D$  of technologies  $t$  and  $u$  on  $D$ , then  $v$  is a technology.

**Proof:** Take  $x^v = (x^v|_{I^t}, x^v|_{I^u \setminus D}) \in X^v$  arbitrary. Then, there exists  $z^t \in Y^t$  such that  $(x^v|_{I^t}, z^t) \in S^t$ , because  $t$  is a technology. Moreover, for  $(z^t|_D, x^v|_{I^u \setminus D})$ , there exists  $z^u \in Y^u$  such that  $((z^u|_D, x^v|_{I^u \setminus D}), ) \in S^u$ , since  $u$  is a technology. Thus, by the definition of the connection  $tu|_D$  of  $t$  and  $u$  on  $D$ ,  $y^v = (z^t|_{O^t \setminus D}, z^u)$  satisfies that  $(x^v, y^v) \in S^v$ .

Take  $y^v = (y^v|_{O^t \setminus D}, y^v|_{O^u}) \in Y^v$  arbitrary. Then, there exists  $z^u \in X^u$  such that  $(z^u, y^v|_{O^u}) \in S^u$ , since  $u$  is a technology. Moreover, for  $y^v = (y^v|_{O^t \setminus D}, z^u|_D)$ , there exists  $z^t \in X^t$  such that  $(z^t, (y^v|_{O^t \setminus D}, z^u|_D)) \in S^t$ , because  $t$  is a technology. Thus, by the definition of the connection  $tu|_D$  of  $t$  and  $u$  on  $D$ ,  $x^v = (z^t, z^u|_{I^u \setminus D})$  satisfies that  $(x^v, y^v) \in S^v$ .

### CONCLUSIONS

This paper gave a mathematical framework for dealing with technologies. The concepts of technologies and their connections were newly provided, and the fact that a connection of technologies satisfies the conditions to be a technology was verified. The framework constructed in this paper allows us to develop such concepts as ‘networks of technologies,’ ‘fungibility of technologies’ and ‘commercializability of technologies.’

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These concepts contribute to develop a mathematical method for evaluating technologies, in particular, patents (Razgaitis (1999), Smith and Parr (1994)). The next step of this research must be defining these concepts rigorously within the framework newly developed in this paper.

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