The number of bankruptcies in Japan fluctuates greatly over time, and the characteristics of these fluctuations have not been investigated sufficiently. This paper examines the trends in bankruptcies in Japan, with a particular focus on identifying the dynamic features of the series. Using quarterly data from 1975 to 2005, we estimate a vector autoregression (VAR) system comprised of four economic variables—the Tokyo Stock Price Index (TOPIX), the break-even point ratio, the debt–equity ratio and the quick assets ratio—and the bankruptcy rate. The VAR is a simple but widely used approach for identifying the path and depth of interrelationships among economic time-series variables. Then we construct impulse response functions, which enable us to assess the dynamic features of the system. In general, the results show expected and consistent relationships between shocks on economic variables and bankruptcies. In particular, a shock on TOPIX has a large effect on bankruptcies. The results further indicate that the movement of the bankruptcy rate reflects the accumulated impact of various shocks in the present and past, to which bankruptcy has distinct response structures.

Keywords: vector autoregression, bankruptcy rate, impulse response, Japan.
In this paper, we attempt to develop a better understanding of this situation. That is, we formally investigate the trends in bankruptcies in Japan, with a particular focus on identifying the dynamic features of the series. For this purpose, we intend to express the economic structure as a vector autoregression (VAR) system. The VAR is a simple but widely used approach for identifying the path and depth of interrelationships among economic time-series variables. Then we construct impulse response functions, which enable us to assess the dynamic features of the system.

In the following sections, we first provide a short description of the VAR system. Then we proceed to explain the data to be analyzed, and show the empirical results. Finally, we discuss the results.

**Figure 1. Number of Bankruptcies in Japan**

The Vector Autoregression (VAR) system has been popular in applied econometrics since Sims’s (1980) seminal work. The standard (reduced-form) VAR is:

$$ y_t = A_0 + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + \epsilon_t, $$

where $y_t$ is an $(n \times 1)$ vector of time-series variables to be analyzed, $p$ is the lag order of the VAR, $A_0$ is an $(n \times 1)$ vector of constants, and $A_k$ are $(n \times n)$ matrices of autoregressive coefficients for $k=1,2,\ldots,p$. The $\epsilon_t$ is an $(n \times 1)$ vector of nonautocorrelated disturbances with zero means and positive definite covariance matrix $E[\epsilon_t \epsilon_t'] = \Omega$. All components of the vector $A_0$ and matrices $A_1,\ldots,A_p$ can be separately estimated by ordinary least squares (see also Sims et al., 1990).

When the stability condition is met (Hamilton, 1994, p. 259), by repeated substitutions, we can obtain the particular solution for the vector $y_t$ as the function of the current and lagged variables of the disturbance vector $\epsilon$. For example, in the simple bivariate ($n=2$) and first-order ($p=1$) case, the VAR is expressed as:

$$ y_t = A_0 + A_1 y_{t-1} + \epsilon_t, $$

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2 The brief explanation in this section is based on Sims (1980), Hamilton (1994), and Enders (2004). Please refer to them for more detailed descriptions.
where \( y_t = \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} \), \( A_0 = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} \), \( A_t = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \), and \( e_t = \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \).

With the stability condition, we can rewrite it as:

\[
y_t = \mu + \sum_{s=0}^{\infty} A^s e_{t-s} ,
\]

where \( \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \), \( \bar{y}_1 = \frac{a_{10}(1-a_{22})+a_{20}a_{12}}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} \), and \( \bar{y}_2 = \frac{a_{20}(1-a_{11})+a_{10}a_{21}}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} \). This is called the infinite moving average representation of the VAR.

Here, we can see the matrix \( A_t^s \) has the interpretation:

\[
\frac{\partial y_{t+s}}{\partial e_{t}} = A_t^s .
\]

In general, a plot of the \((i, j)\) element of \( A_t^s \), that is, \( \frac{\partial y_{t+s}}{\partial e_{t}} \) as a function of \( s \), is called the impulse-response function. It describes the response of \( y_{t+s} \) to a one-time impulse in \( y_t \). As briefly illustrated above, the functions can be calculated by the estimated parameters. In fact, plotting them is a practical and popular way to visually show the dynamic behavior of the system in response to the various shocks.

The core idea of the VAR is that all variables are (potentially) treated as endogenous variables, and the relationships between them are unrestricted before estimation. Sims (1980) proposed the model as having advantages over the large-scale structural equation models. Since then, the approach has been widely applied for capturing economic systems and investigating their dynamic behaviors.

**VARIABLES**

First, we use the variable of the bankruptcy rate, which is defined by \([\text{the number of bankruptcies of corporations in the period}] / [\text{the number of corporations at the end of the previous period}]\). The number of corporations has varied (i.e., increased) in Japan, and thus the adjustment for evaluating the number of bankruptcies in comparison with the total number of corporations makes sense. We exclude individual proprietorships, mainly because we use the results of the survey of corporations as stated below.\(^3\) The data on the number of corporations is obtained from the results of the National Tax Agency Annual Statistics Reports by the National Tax Agency.\(^4\)

Figure 2 shows the bankruptcy rate.\(^5\) For the estimation, we use quarterly data from the second quarter of 1975 to the first quarter of 2005 (i.e., 120 quarters in total). We seasonally adjusted the data using X-12 ARIMA.

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\(^3\) Individual proprietorships account for about 20% of the total number of bankruptcies. They are included in Figure 1.

\(^4\) As the National Tax Agency provides only annual data, we linearly interpolate these data for quarterly analysis.

\(^5\) The shadows in Figures 2 and 3 indicate recession periods, as officially published by the Economic and Social Research Institute (ESRI), Cabinet Office.
Other than the bankruptcy rate, we use four economic variables: the Tokyo Stock Price Index (TOPIX), the break-even point ratio, the debt–equity ratio, and the quick assets ratio. As well as the bankruptcy rate, quarterly data from 1975 Q2 to 2005 Q1 are used. First, TOPIX (provided by the Tokyo Stock Exchange) is adopted to represent the general business environment, and possibly to act as a proxy of the business confidence of the economy.

The three remaining variables are constructed from the results of the Financial Statements Statistics of Corporations by Industry, Quarterly (by the Ministry of Finance). It surveys quarterly financial statements of corporations (except for those of the finance and insurance industry), the paid up capital of which is greater than or equal to 10 million yen, and publishes the aggregated results, classified by industry and the amount of paid-up capital. It is an authorized and official series of statistics in Japan, and widely used for economic analyses.

The break-even point ratio is adopted to represent the profitability of corporations. It is conceptually defined as \([\text{break-even point sales}] / [\text{the actual sales}]\), with a high ratio corresponding to low profitability of the business unit, whereas a low ratio corresponds to high profitability. To be more specific, the break-even point ratio is constructed by \((\text{fixed cost} / \text{marginal profit rate}) / \text{sales}\), where \(\text{fixed cost} = \text{labor cost} + \text{interest cost and discount charge} + \text{depreciation cost}\); the marginal profit rate \(= (\text{sales} − \text{variable cost}) / \text{sales}\); and the variable cost \(= \text{sales} − \text{fixed cost} − \text{ordinary profit}\). The series and its definition are relatively popular; for example, they are also used in analyses in recent white papers by the government (Cabinet Office, 2005 and 2006).

The debt–equity ratio is a variable representing the financial strength in the medium and long term. It is defined as \([\text{total amount of debt}] / [\text{total amount of equity}]\). In principle, a high ratio indicates the possibility of over-borrowing or a low equity situation, which indicates that the business unit is in an unsound condition. Finally, the quick assets ratio represents the financial soundness and solvency in the immediate or very short term. A high ratio indicates that there is enough capacity to pay at the time, which is associated with a sound business unit. It is defined as \([\text{the quick assets}] / [\text{the current liabilities}]\).

6 Thus, the marginal profit rate can be rewritten as \((\text{fixed cost} + \text{ordinary profit}) / \text{sales}\).
amount of quick assets] / [the amount of liquid liabilities], where the quick assets = cash and deposits + bills and accounts receivable + short term securities; and the liquid liabilities = bills and accounts payable + short-term borrowings + allowance + other liquid liabilities. It is hypothesized that these variables have some possible relationship to bankruptcies, even at the level of aggregated data.

Figure 3 shows the four variables. We seasonally adjusted the three variables other than TOPIX using X-12 ARIMA. TOPIX is used by taking the natural logarithm of the (not seasonally adjusted) series in estimation, although Figure 3 shows the original series before taking the logarithm.

**Figure 3. Variables**

**ESTIMATION AND IMPULSE RESPONSES**

Under the five variables, we estimate a vector autoregression (VAR) system. The lag-order is set by four, because we are undertaking quarterly analyses. We estimate two types of VAR. The first is a level estimation, in which the set of these variables are directly used as a vector $y_t$. The second is a differencing estimation, in which the set of the first-differences of these variables are used as a vector $y_t$. To identify the impulse responses, we use generalized impulse-response functions (Pesaran and Shin, 1998).

Figure 4 shows the result of the response functions in the level estimation. Figure 5 shows the result of the response functions in the differencing estimation. Both estimations show the results of the bankruptcy rate up to 20 quarters after a generalized one standard deviation shock on each of the four economic variables. In
Bankruptcy Dynamics in Japan

Figure 5, we report the accumulated responses to enable a direct comparison of the results of the differencing estimation with the results of the level estimation.

In general, the figures show expected and consistent relationships between the shocks on economic variables and the bankruptcies: that is, the positive shocks on TOPIX and the quick assets ratio work to depress the bankruptcy rate, and the shocks on the break-even point ratio and the debt–equity ratio act to raise the bankruptcy rate. In that sense, our expression of the economic system and its estimation seems to succeed. In particular, it appears that the shock on TOPIX has a large effect on the bankruptcy rate, whereas the effect of a shock on the debt–equity ratio is relatively small (in Figure 5) or unclear (in Figure 4).

Response to Generalized One S.D. Innovations ± 2 S.E.

Figure 4. Impulse Responses of Bankruptcy Rate: Level Estimation
DISCUSSION

We estimated a VAR system comprised of four economic variables and the bankruptcy rate, using quarterly data from 1975 to 2005. As the result of the impulse responses, a convincing relationship between shocks on economic variables and bankruptcies is shown. Our results further indicate that the movement of the bankruptcy rate reflects the accumulated impact of various shocks in the present and past, to which bankruptcy has distinct response structures. Although the formula and results reported in this paper remain simple, we could utilize them as a foundation and extend them for further studies. In particular, it would be worthwhile to incorporate differences among industries and levels of firm size into the analysis. Moreover, some kinds of institutional or legal reforms could be incorporated. In parallel, our approach may make a contribution to an empirical model of the Japanese business cycle.

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